General Relativity and Einstein's Gravity

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Philosophical Principles of Relativity

Great Relativists

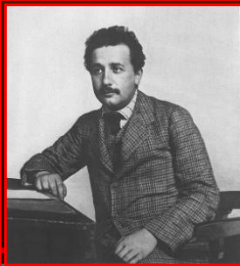
Gottfried Leibniz (1646-1716)

0. *Space is relative and must be thought of as a set of relationships between material objects.*

Ernst Mach (1938-1916)

1. *Absolute space is absurd because it is unobservable*

2. *Our principles of mechanics are experimental knowledge concerning relative positions and motions of bodies*



3. *All physical reference frames must be connected with bodies and should be physically equivalent*

Generalization of the Galilei-Newton Transformations

The leader

Joseph Larmor (1857-1942)

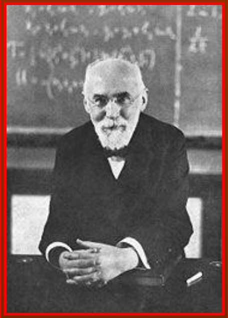
- In 1897 J. Larmor proved the invariance of free Maxwell equations using coordinates and fields transformations

$$x' = (x - vt)\beta,\ y' = y,\ z' = z,\ t' = (t - \frac{xv}{c^2})\beta,\quad \beta = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$E'_x = E_x,\ E'_y = (E_y - \frac{v}{c} H_z)\beta,\ E'_z = (E_z + \frac{v}{c} H_y)\beta,$$

$$H'_x = H_x,\ H'_y = (H_y - \frac{v}{c} E_z)\beta,\ H'_z = (H_z + \frac{v}{c} E_y)\beta$$

- In 1904 H. Lorentz used this transformations for proving the invariance of Maxwell equations with sources (he made a mistake).
- In 1905 A. Einstein and H. Poincare have published the correct proof.
- H. Poincare and A. Einstein have named these transformations as Lorentz transformations.



Hendrik Lorentz (1853-1928)

Henri Poincare (1854-1912)

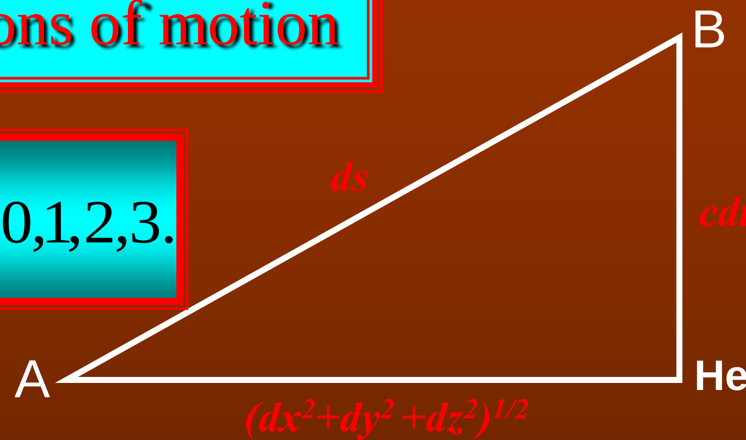
Special Relativity is the First Generalization of Newton's Mechanics

The leader

(1905)

Relativistic equations of motion

$$m\frac{d^2x^i}{ds^2} = F^i, \; i = 0,1,2,3.$$



Hermann Minkowski (1864-1909)

- The Laws of Physics are the same for all inertial Observers (frames of constant velocity)
- The speed of light, c, is a constant for all inertial Observers
  - → Events are characterized by 4 coordinates (**ct,x,y,z**)
  - → Length Contraction, Time Dilation, Mass increase
  - → Space and Time are linked
    - The notion of ***SPACE-TIME***

The Minkowski Metric :

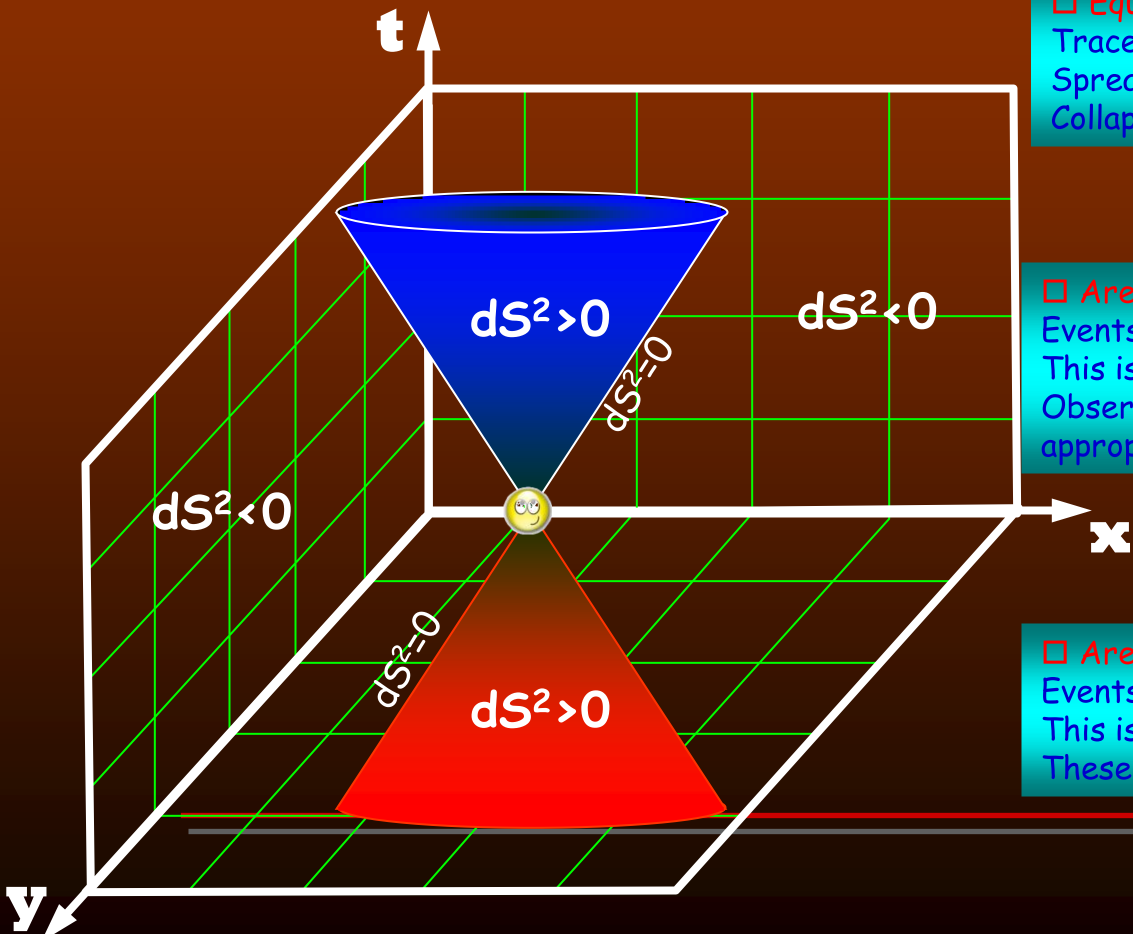
$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$$

$$ds^2 = c^2dt^2 - \left[dr^2 + r^2d\theta^2 + r^2 Sin^2\theta d\phi^2\right]$$

The Minkowski Space-Time

Causally Connected Events in Minkowski Space-time

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$$



- Equation of a light ray, $dS^2=0$, $\frac{dx}{dt} = \pm c$
  Trace out light cone from Observer in Minkowski S-T
  Spreading into the future
  Collapsing from the past

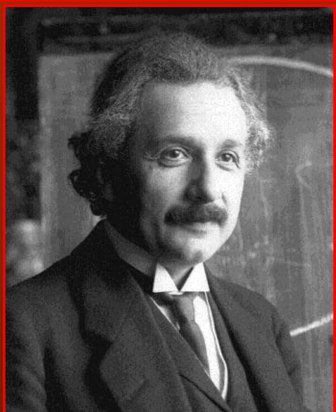
- Area within light cone: $dS^2>0$
  Events that can affect observer in past,present,future.
  This is a timelike interval.
  Observer can be present at 2 events by selecting an appropriate speed.

- Area outside light cone: $dS^2<0$
  Events that are causally disconnected from observer.
  This is a spacelike interval.
  These events have no effect on observer.

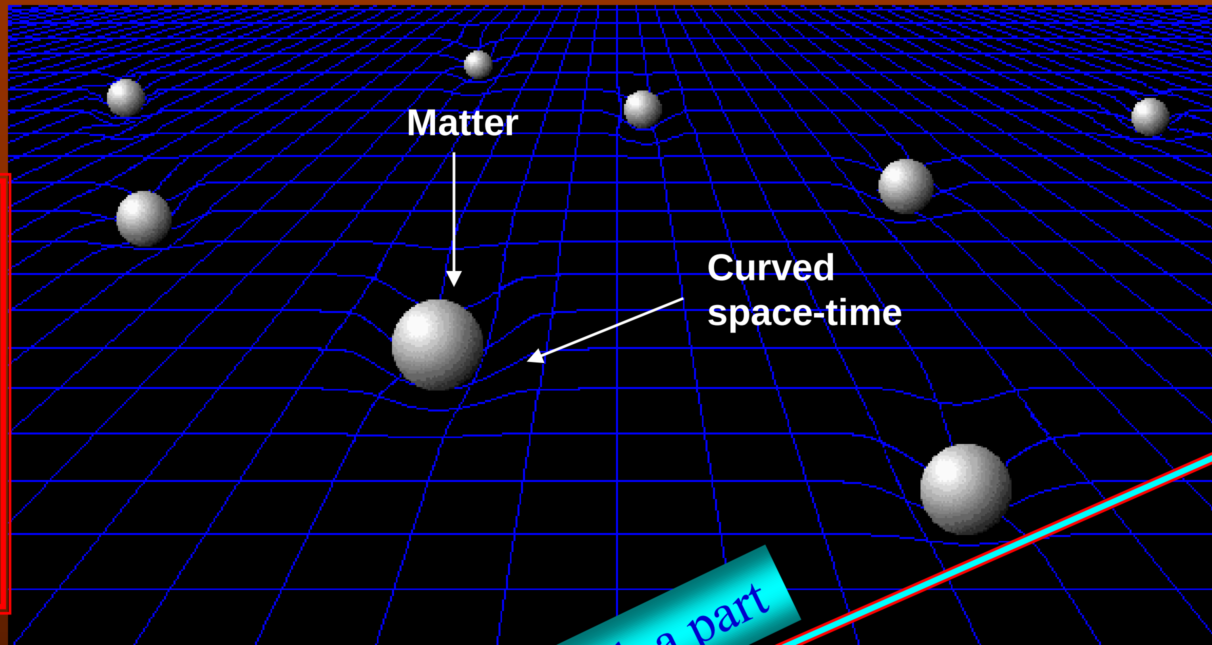
Einstein's Theory of Gravity

(1907-1915)

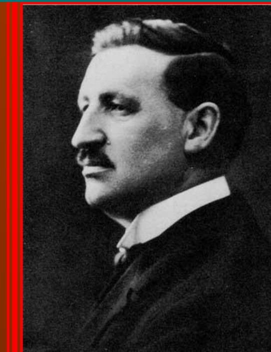
The leader



Albert Einstein



Friend of Einstein



Marcel Grossman

David Hilbert

Took a part

1913

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds}=0$$

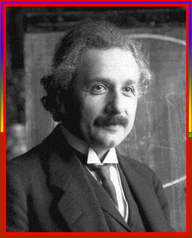
4 Equations of motion

1915

$$G_{ik}=R_{kl}-\frac{1}{2}g_{ik}R=\frac{8\pi G}{c^4}T^{ik}$$

10 Field equations

Took a part



Physical Principles

Galileo Galilei (1564-1642)

1.Weak Equivalence Principle

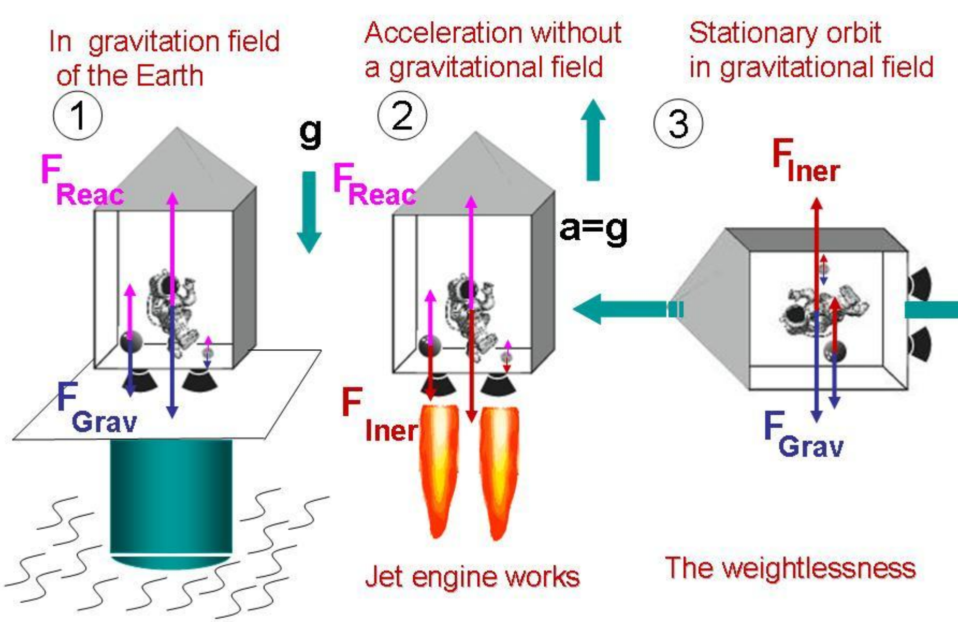
$$m_g = m_i$$

A more general interpretation led Einstein to his theory of General Relativity

2. STRONG PRINCIPLE OF EQUIVALENCE:

An observer cannot distinguish between a local gravitational field and an equivalent uniform acceleration

Experience and Strong Principle of Equivalence



- Imagine an astronaut in spaceship
- the first spaceship is standing on Earth
- the second being accelerated with a=g
- the third moves in gravitational field
- For 1 case a gravitational force and supporting force acts on astronaut.
- For 2 case a force of inertia and supporting force acts on astronaut.
- For 3 case two forces which compensate each other act on astronaut.

Accelerated Local Lorenz reference frame:

- Locally objects move freely – space Euclidean.
- Globally space is not Euclidean !

The Interval in General Relativity

Enlightenment

The interval is given by:

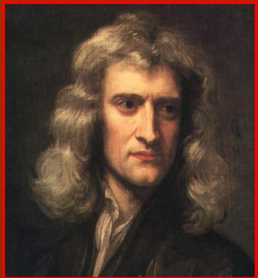
$$\boxed{ds^2 = \sum_{i,j=0}^{n} g_{ij}\, dx^i dx^j}$$

The interval ds depends on gravitational potential !!!

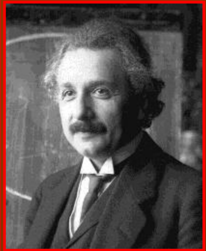
- $\boxed{g_{ij}}$ is the *metric tensor that:*
- Tells us how to calculate the distance between 2 points in any given space -time
- Components of $\boxed{g_{ij}}$ Multiplicative factors of differential displacements (dx^i)
  - Generalized Pythagorean Theorem

Marcel Grossman, Einstein,
Gustav Geissler, and Eugen Grossman. Marcel Grossman,
whom Einstein met in Zurich, quickly recognized his friend's genius.
He did all he could to promote Einstein's career.

The Equations of Motion



Einstein c.1916: "I have made a great discovery in mathematics; I have suppressed the summation sign every time that the summation must be made over an index which occurs twice..."



Interval

$$ds = \sqrt{g_{ik}dx^i dx^k}$$

$$\delta \int_{path} ds = 0$$

But lines are not straight (because of the metric tensor)

Gravitational force

Gravity is a property of Spacetime, which may be curved

Path of a free particle is a geodesic

$$m\frac{d^2x^i}{ds^2} + m\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0$$

Force of inertia

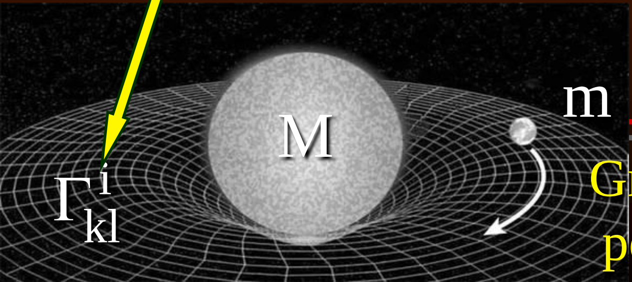
1 → Intensity of the gravitational field

$$\Gamma^i_{kl} = \frac{g^{im}}{2}\left(g_{mk,l} + g_{ml,k} - g_{kl,m}\right)$$

$$g_{00} = \left(1 + \frac{2\varphi_N}{c^2}\right)$$

When $\left|\frac{2\varphi_N}{c^2}\right| << 1$

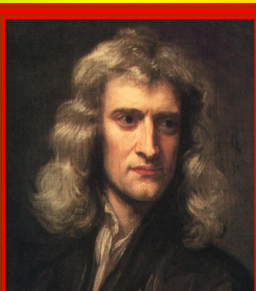
Newton like equations



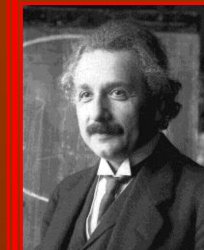
Gravitational potential

$$m\frac{d^2x^\alpha}{dt^2} = -m\Gamma^\alpha{}_{00} = m\frac{MG}{r^3}x^\alpha, \quad \alpha = 1,2,3$$

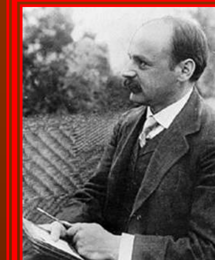
The Field Equations



$$R_{kl} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T^{ik}$$



$$R_{ik} = 0$$



Schwarzschild Solution (1916)

Karl Schwarzschild (1873-1916)

$$ds^2 = (1 - \frac{r_g}{r}) c^2 dt^2 - (1 - \frac{r_g}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

When $\frac{r_g}{r} << 1$

Satang

$$r_g = \frac{2MG}{c^2}$$

Gravitational radius

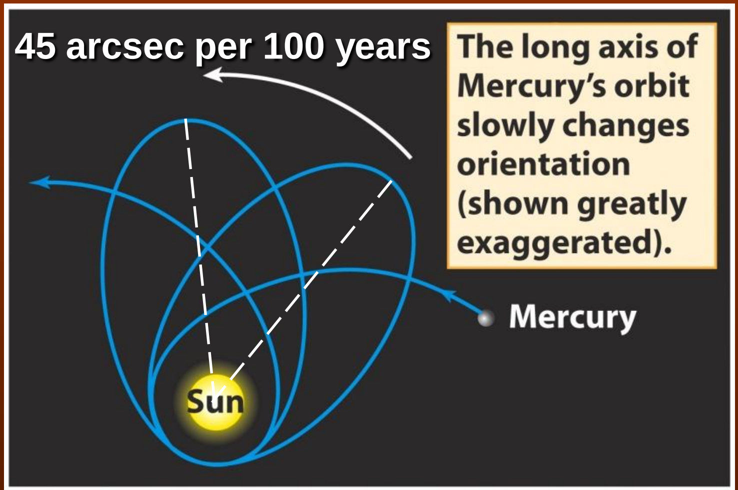
$$\Delta \varphi = 4\pi\rho$$

9mm

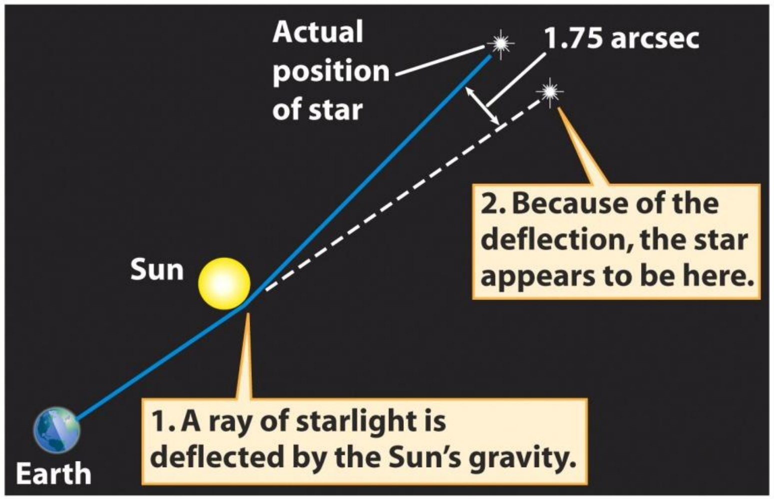
Gravitational radius of the Earth

Comparison with experiment.

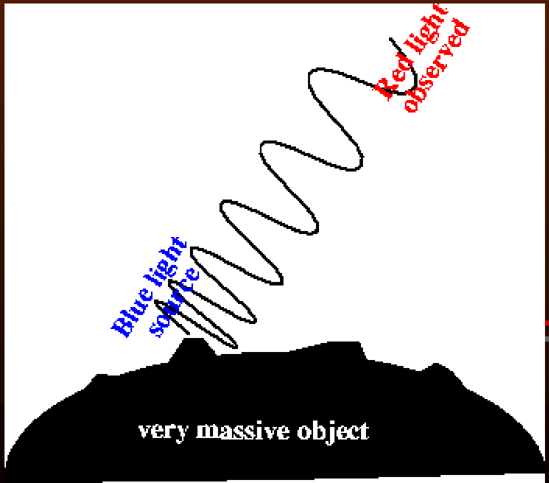
1. Precession of Mercury's orbit



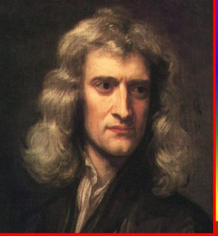
2. Gravity bends the path of light



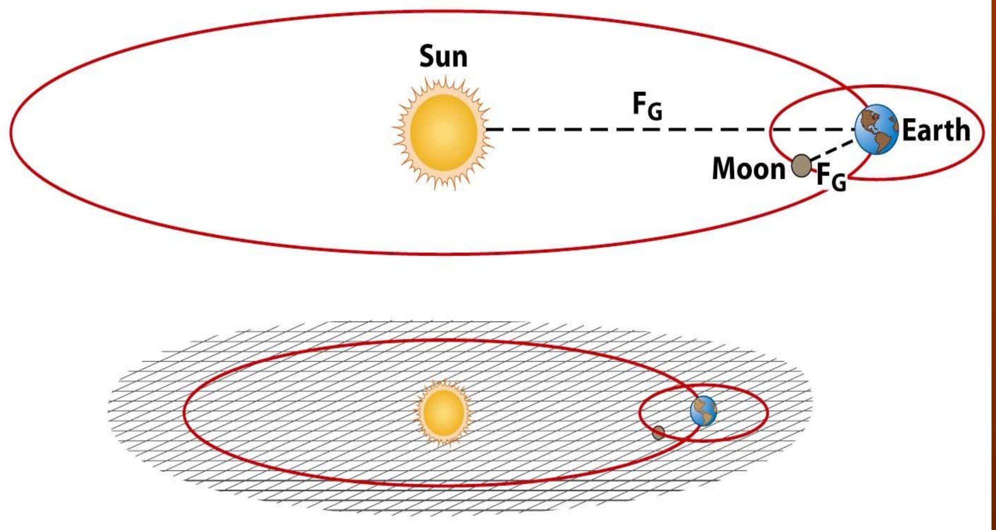
3. Gravitational Curvature of time



Clocks on first floor tick more slowly than clocks on top of the building
(roughly 1 s per 3 x 106 years).



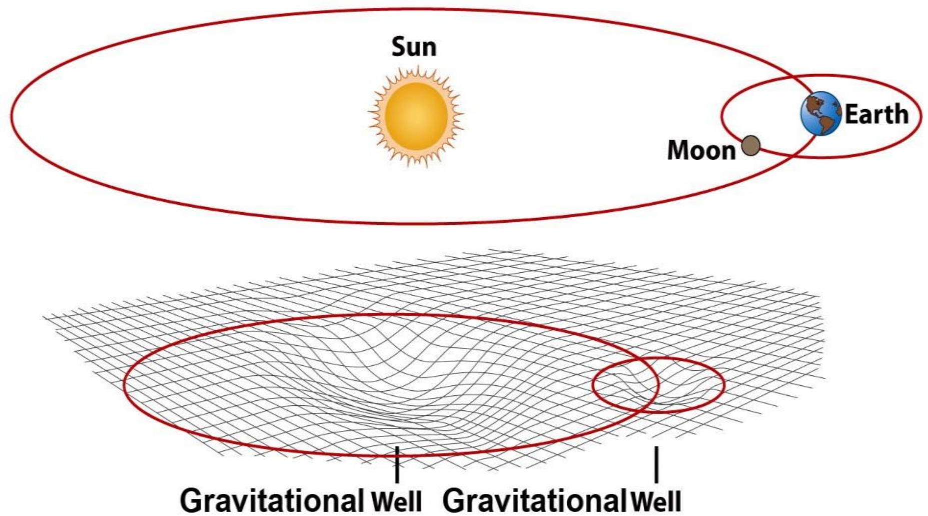
General Relativity is the Second Generalization of Newton's Mechanics



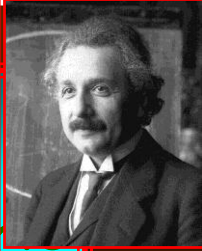
Newton: objects move in the 3D flat space under action of forces

Newton:
- Mass tells Gravity how to make a Force
- Force tells mass how to accelerate

Newton:
- Flat Euclidean Space
- Universal Frame of reference



Einstein: objects move in 4D space-time curved by a matter

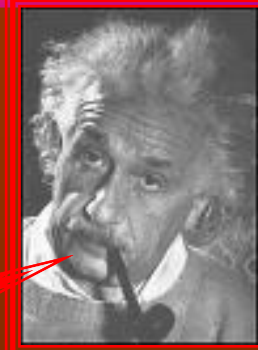


Einstein:
- Mass tells space how to cu
- Space tells mass how to move

Einstein:
- Space can be curved
- It's all relative anyway!!

The main Problem

$$R_{ij} - \frac{1}{2} R\, g_{ij} = \frac{8\pi G}{c^4} T_{ij}$$



The right hand side includes all those that cannot be described so fare in the unified field theory. Such a formulation is just a temporary answer, undertaken to give general relativity some accomplished expression. That theory of the gravitation field is separated in somewhat artificial manner from the Unified Field of yet unknown nature.

Lovelock's theorem

For Einstein's equations

$$a = \frac{8\pi G}{c^4}, \qquad b = 1$$

David Lovelock

Any non-geometrical energy-momentum tensor in right hand side of the Einstein's equations does not define geometry of the surrounding space-time

$$bG_{ik} = aT_{ik}$$

When

$$G_{ik} = 0$$

$$b \neq 0, a \neq 0$$

we always have

$$T_{ik} = 0$$

$$G_{ik} = R_{ik} - \frac{1}{2} g_{ik} R$$

except in a case

$$G_{ik} = -\Lambda g_{ik}$$

The summary on the General Relativity and Einstein's Gravity

- Special Relativity is the first generalization of Newton's Mechanics.

- General Relativity is the second generalization of Newton's Mechanics.

- Only Schwarzschild solution of the Vacuum Einstein's equations has got experimental verification.

- The completed Gravitation Theory demands geometrization of the Energy-Momentum Tensor in right hand side of Einstein's equations.

To be continued by General Relativity 2

Kob Khun Krab!

Thank You for Your Attention !