General Relativity
and Einstein's Gravity

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1. All physical reference frames must be connected with bodies and should be physically equivalent.

2. Our principles of mechanics are experimental knowledge concerning relative positions and motions of bodies.

3. Absolute space is absurd because it is unobservable.

4. Space is relative and must be thought of as a set of relationships between material objects.

Gottfried Leibniz (1646-1716)

Ernst Mach (1838-1916)
In 1897 J. Larmor proved the invariance of free Maxwell equations using coordinates and fields transformations:

\[ x' = (x - vt)\beta, \quad y' = y, \quad z' = z, \quad t' = (t - \frac{xv}{c^2})\beta, \quad \beta = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \]

\[ E'_x = E_x, \quad E'_y = (E_y - \frac{v}{c}H_z)\beta, \quad E'_z = (E_z + \frac{v}{c}H_y)\beta, \]

\[ H'_x = H_x, \quad H'_y = (H_y - \frac{v}{c}E_z)\beta, \quad H'_z = (H_z + \frac{v}{c}E_y)\beta \]

In 1904 H. Lorentz used this transformations for proving the invariance of Maxwell equations with sources (he made a mistake).

In 1905 A. Einstein and H. Poincare have published the correct proof.

H. Poincare and A. Einstein have named these transformations as Lorentz transformations.

The leader

Joseph Larmor (1857-1942)

Hendrik Lorentz (1853-1928)

Henri Poincare (1854-1912)
The Laws of Physics are the same for all inertial Observers (frames of constant velocity)
• The speed of light, $c$, is a constant for all inertial Observers
  → Events are characterized by 4 coordinates $(ct, x, y, z)$
  → Length Contraction, Time Dilation, Mass increase
  → Space and Time are linked
  ➞ The notion of **SPACE-TIME**

### Special Relativity is the First Generalization of Newton’s Mechanics

**The Minkowski Metric**:

\[ ds^2 = c^2 dt^2 - \left( dx^2 + dy^2 + dz^2 \right) \]

\[ ds^2 = c^2 dt^2 - \left[ dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \]

$\text{Hermann Minkowski}$

(1864-1909)

$\text{(1905)}$
Causally Connected Events in Minkowski Space-Time

\[ ds^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2) \]

- **Equation of a light ray, \( dS^2 = 0 \):**
  - Trace out light cone from Observer in Minkowski S-T
  - Spreading into the future
  - Collapsing from the past

- **Area within light cone: \( dS^2 > 0 \)**
  - Events that can affect observer in past, present, future.
  - This is a *timelike* interval.
  - Observer can be present at 2 events by selecting an appropriate speed.

- **Area outside light cone: \( dS^2 < 0 \)**
  - Events that are causally disconnected from observer.
  - This is a *spacelike* interval.
  - These events have no effect on observer.

\[ \frac{dx}{dt} = \pm c \]
Einstein's Theory of Gravity

(1907-1915)

The leader

Matter

Curved space-time

\[ \frac{d^2 x^i}{ds^2} + \Gamma^i_{kl} \frac{dx^k}{ds} \frac{dx^l}{ds} = 0 \]

\[ G_{ik} = R_{kl} - \frac{1}{2} g_{ik} R = \frac{8 \pi G}{c^4} T_{ik} \]

4 Equations of motion

10 Field equations

Friend of Einstein

Took a part

Albert Einstein

Marcel Grossman

David Hilbert

Took a part

(1907-1915)
1. Weak Equivalence Principle

A more general interpretation led Einstein to his theory of General Relativity.

\[ m_g = m_i \]

2. STRONG PRINCIPLE OF EQUIVALENCE:

An observer cannot distinguish between a local gravitational field and an equivalent uniform acceleration.

**Experience and Strong Principle of Equivalence**

- Imagine an astronaut in spaceship
  - the first spaceship is standing on Earth
  - the second being accelerated with \( a = g \)
  - the third moves in gravitational field

- For case 1: a gravitational force and supporting force acts on astronaut.
- For case 2: a force of inertia and supporting force acts on astronaut.
- For case 3: two forces which compensate each other act on astronaut.

Accelerated Local Lorentz reference frame:
- Locally objects move freely - space Euclidean.
- Globally space is not Euclidean!
The interval is given by:

\[ ds^2 = \sum_{i,j=0}^{n} g_{ij} dx^i dx^j \]

- \( g_{ij} \) is the metric tensor that:
  - Tells us how to calculate the distance between 2 points in any given space-time
  - Components of \( g_{ij} \) are multiplicative factors of differential displacements \((dx^i)\)
  - Generalized Pythagorean Theorem

The interval \( ds \) depends on gravitational potential !!!

Marcel Grossman, Einstein, Gustav Geissler, and Eugen Grossman. Marcel Grossman, whom Einstein met in Zurich, quickly recognized his friend's genius. He did all he could to promote Einstein's career.
The Equations of Motion

Interval

\[ ds = \sqrt{g_{ik} \, dx^i \, dx^k} \]

\[ \delta \int_{\text{path}} ds = 0 \]

But lines are not straight (because of the metric tensor)

Gravitational force

\[ m \frac{d^2 x^i}{ds^2} + m \Gamma^i_{kl} \frac{dx^k}{ds} \frac{dx^l}{ds} = 0 \]

Force of inertia

Intensity of the gravitational field

\[ g_{00} = 1 + \frac{2\varphi_N}{c^2} \]

\[ \Gamma^i_{kl} = \frac{g^{im}}{2} \left( g_{mk,l} + g_{ml,k} - g_{kl,m} \right) \]

Newton like equations

\[ m \frac{d^2 x^\alpha}{dt^2} = -m \Gamma^\alpha_{00} = m \frac{MG}{r^3} x^\alpha, \quad \alpha = 1, 2, 3 \]

Gravity is a property of Spacetime, which may be curved

Path of a free particle is a geodesic

Einstein c.1916: "I have made a great discovery in mathematics; I have suppressed the summation sign every time that the summation must be made over an index which occurs twice..."
The Field Equations

\[ R_{kl} - \frac{1}{2} g_{ik} R = \frac{8 \pi G}{c^4} T^{ik} \]

Schwarzschild Solution (1916)

\[ ds^2 = (1 - \frac{r_g}{r})c^2 dt^2 - (1 - \frac{r_g}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ r_g = \frac{2 MG}{c^2} \]

Gravitational radius

\[ \Delta \phi = 4 \pi \rho \]

Gravitational radius of the Earth

9mm
Comparison with experiment.

1. Precession of Mercury’s orbit
   - 45 arcsec per 100 years
   - The long axis of Mercury’s orbit slowly changes orientation (shown greatly exaggerated).

2. Gravity bends the path of light
   - 1.75 arcsec
   - Because of the deflection, the star appears to be here.
   - 1. A ray of starlight is deflected by the Sun’s gravity.

3. Gravitational Curvature of time
   - Clocks on first floor tick more slowly than clocks on top of the building
   - (roughly 1 s per 3 x 10^6 years).
General Relativity is the Second Generalization of Newton’s Mechanics

Newton:
- Mass tells Gravity how to make a Force
- Force tells mass how to accelerate
- Flat Euclidean Space
- Universal Frame of reference

Einstein:
- Mass tells space how to curve
- Space tells mass how to move
- Space can be curved
- It’s all relative anyway!!
The main Problem

\[ R_{ij} - \frac{1}{2} R g_{ij} = \frac{8 \pi G}{c^4} T_{ij} \]

The right hand side includes all those that cannot be described so far in the unified field theory. Such a formulation is just a temporary answer, undertaken to give general relativity some accomplished expression. That theory of the gravitation field is separated in somewhat artificial manner from the Unified Field of yet unknown nature.

Lovelock's theorem

Any non-geometrical energy-momentum tensor in right hand side of the Einstein's equations does not define geometry of the surrounding space-time.

For Einstein's equations

\[ a = \frac{8\pi G}{c^4}, \quad b = 1 \]

\[ b G_{ik} = a T_{ik} \]

When we always have except in a case

\[ G_{ik} = 0 \]

\[ T_{ik} = 0 \]

\[ G_{ik} = -\Lambda g_{ik} \]
The summary on the General Relativity and Einstein's Gravity

- Special Relativity is the first generalization of Newton’s Mechanics.

- General Relativity is the second generalization of Newton’s Mechanics.

- Only Schwarzschild solution of the Vacuum Einstein’s equations has got experimental verification.

- The completed Gravitation Theory demands geometrization of the Energy-Momentum Tensor in right hand side of Einstein’s equations.
To be continued by General Relativity 2
Kob Khun Krab!

Thank You for Your Attention!