FOUNDATION OF THE MECHANICS OF ORIENTABLE POINT

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The mechanics of an orientable point (point with "spin") based on 3D and 4D Frenet equations is considered. In such mechanics there is an opportunity to describe formally any physical trajectory of a particle with own rotation. We use anholonomic rotational coordinates (Euler angles) as elements of internal space of the mechanics which generate a rotational relativity. The groups of transformations of the mechanics of an orientable point form Poincare's group with semidirect product of translations and rotations, so translational and rotational momentums appear dependent from each other. Connection of the curve torsion with Ricci rotational coefficients is shown and rotational metric is entered. Equivalence between equations of motion 4D oriented point and geodesic equations of absolute parallelism geometry is established. The space of events an arbitrary accelerated 4D frame of reference, which has 10 degrees of freedom, is described by Cartan structure equations of absolute parallelism geometry $A_4(6)$. It represent 10D coordinate space in which 4 translational coordinates $x_0 = ct,\ x_1 = x,\ x_2 = y, x_3 = z$ describe motion of the origin O 4D orientable point and 6 angular coordinates $\varphi_1 = \varphi,\ \varphi_2 = \psi,\ \varphi_3 = \theta,\ \varphi_4 = \vartheta_x,\ \varphi_5 = \vartheta_y,\ \varphi_6 = \vartheta_z$ describe change of its orientation.

The structural equations of absolute parallelism geometry $A_4(6)$, represent an extended set of Einstein-Yang-Mills equations with the gauge translations group T_4 defined on the base x^i and with the gauge rotational group $O(1.3)$, defined in the fibre $e^i{}_a$. The sources in these equations are defined through the torsion (torsion field) of $A_4(6)$ geometry. The received system of the equations represents generalization vacuum Einstein's equations on a case when sources have geometrical nature. On the basis of the Vaidya-like solution of the Einstein-Yang-Mills equations correspondence with the Einstein's equations is established.

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I. INTRODUCTION

Einstein proposed the following approach to the construction of the fundamental theory of things in the microworld. On the left-hand side of his famous equations

$$R_{jm} - \frac{1}{2} g_{jm} R = \frac{8\pi G}{c^4} T_{jm} \qquad (1)$$

one finds a purely geometrical quantity (the Einstein tensor $G_{jm} = R_{jm} - \frac{1}{2} g_{jm} R$), and on the right-hand side, the energy-momentum tensor T_{jm} for mater, which was, so to speak, introduced "manually." In Einstein's picture, mater appears thus against the background of a curved space-time as an entity independent of space-time.

Einstein was not satisfied with the phenomenological representation of T_{jm}, since: "The right-hand side includes all that cannot be described so far in the unified field theory. Of course, not for a fleeting moment I have had any doubt that such a formulation is just a temporary answer, undertaken to give to general relativity some closed expression. This formulation has been in essence nothing more that the theory of the gravitation field, which has been separated in a somewhat artificial manner from the unified field of a yet unknown nature[1]."

A way to remove arbitrariness in the selection of the energy-momentum tensor was seen by Einstein in the geometrization of the energy-momentum tensor of matter on the right-hand side of Einstein's equations (1). Einstein believed that the geometrization of the energy-momentum tensor of matter should result in the geometrization of the matter field that make it up. For Einstein the geometrization of matter fields implied the construction of a fundamental theory of phenomena in the microworld that is in conformity with relativity principle. 30 years he tried constructing a "reasonable general relativistic theory," and within its framework a "more advanced quantum theory[2]."

In the present work we offer to geometrize the right part of the equations (1), using instead of a material point of general relativistic Einstein's mechanics more the general object - an orientable material point. We shall understand any accelerated reference frame, formed by unit orthogonal vectors as an orientable material point. In 3D translational coordinates space the orientable point has 6 degrees of freedom, in 4D translational coordinate space - ten. Generalization of the Einstein's theory offered by us allows analytically to describe Descartes's approved idea, that any real motion is a rotation.

II. 3D ORIENTED POINT AND GENERALIZATION OF THE EQUATIONS OF NEWTON'S MECHANIC

In 1847 French mathematician Jean F. Frenet in his thesis has written equations, describing motion of oriented point in the 3D space along an arbitrary curve $\mathbf{x} = \mathbf{x}(s)$, where s – the arc length. Equations are written for the three orthogonal unit vectors $\mathbf{t}$, $\mathbf{n}$ and $\mathbf{b}$ with orthogonality conditions $\mathbf{t}^2 = \mathbf{n}^2 = \mathbf{b}^2 = 1$, $\mathbf{tn} = \mathbf{nb} = \mathbf{bt} = 0$. The tangent unit vector $\mathbf{t}$ is choused as tangent to the curve at point M (fig.1), pointing the direction of motion. The normal unit vector $\mathbf{n}$ is the derivative of $\mathbf{t}$ with respect to the arc length parameter of the curve, divided by its length and the binormal unit vector $\mathbf{b}$ is defined as the cross product $\mathbf{b} = \mathbf{t} \times \mathbf{n}$. For these vectors Frenet's equations look like[3]

$$\frac{d\mathbf{x}}{ds} = \mathbf{t} \ . \tag{2}$$

$$\frac{d\mathbf{t}}{ds} = \kappa(s)\mathbf{n} \ , \tag{3}$$

$$\frac{d\mathbf{n}}{ds} = -\kappa(s)\mathbf{t} \ + \chi(s)\mathbf{b} \ , \tag{4}$$

$$\frac{d\mathbf{b}}{ds} = -\chi(s)\mathbf{n} \ , \tag{5}$$

where $\kappa(s)$ - curvature of the curve and $\chi(s)$ - torsion of the curve. Frenet was the first who has shown that arbitrary curve in 3D flat space is determined by two scalar parameters - curvature $\kappa(s)$ and torsion $\chi(s)$.

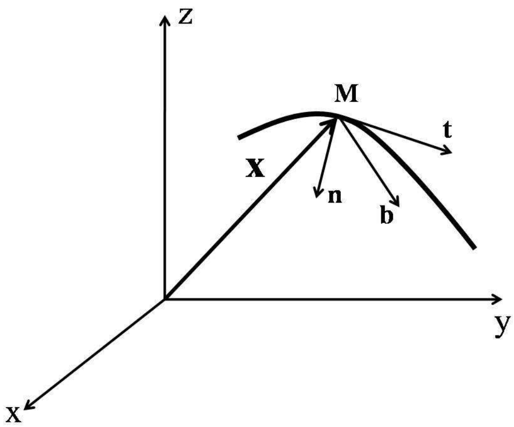

FIG. 1: Trajectory of the 3D oriented point

Differentiating equations (2) on s and using (3)-(5) and orthogonality conditions, we shall get the equations

$$\frac{d^2\mathbf{x}}{ds^2} = \kappa(s)\mathbf{n} \ , \tag{6}$$

$$\frac{d^3\mathbf{x}}{ds^3} = \frac{d\kappa(s)}{ds}\mathbf{n} - \kappa^2(s)\mathbf{t} + \kappa(s)\chi(s)\mathbf{b} \ , \tag{7}$$

describing motion of the triad origan O (motion of point M).

For comparison of the equations (6) (7) with the equations of Newton's mechanics, it is convenient to pass in them to time parameter t

$$\frac{d^2\mathbf{x}}{dt^2} = a\mathbf{t} + \kappa v^2\mathbf{n} \ , \tag{8}$$

$$\frac{d^3\mathbf{x}}{dt^3} = (\frac{da}{dt} - \kappa^2 v^3)\mathbf{t} + (3av\kappa + v^2\frac{d\kappa}{dt})\mathbf{n} + \kappa\chi v^3\mathbf{b} \ , \tag{9}$$

where $v = ds/dt$ – absolute velocity and $a = dv/dt$– – tangent acceleration. Multiplying these equations on mass m, we shall receive the translational equations of motion of an orientable point with the law of transformation infinitesimal vector dx_α

$$dx_{\alpha'} = \frac{\partial x_{\alpha'}}{\partial x_\alpha}dx_\alpha, \quad \alpha = 1, 2, 3,$$

where matrixes $\partial x_{\alpha'}/\partial x_\alpha$ form the group 3D translations $T(3)$. The equations (8) are similar to the equations of Newton mechanics, but have geometrical nature. A choice of curvature κ and parameter s it is possible to describe formally any physical trajectory of a particle in 3D space, moving under action of force $\mathbf{F} = m(a\mathbf{t} + \kappa v^2\mathbf{n})$. The equations (9) have no analogues in the Newton mechanic as contain the third derivative of coordinate on time. In electrodynamics we known equations of motion of radiating charge

$$m\ddot{\mathbf{x}} = e\mathbf{E} + \frac{e}{c}[\dot{\mathbf{x}}\mathbf{H}] + \frac{2e^2}{3c^3}\dddot{\mathbf{x}} \ , \tag{10}$$

which contain the third derivative of coordinate on time. Using (9) we have for reaction force of the radiation we have

$$\mathbf{F}_{rad} = \frac{2e^2}{3c^3}\{(\frac{da}{dt} - \kappa^2 v^3)\mathbf{t} + (3av\kappa + v^2\frac{d\kappa}{dt})\mathbf{n} + \kappa\chi v^3\mathbf{b}\}. \tag{11}$$

From these equations one can see that the reaction force of the radiation in electrodynamics has complex structure. It contains terms generated not only by external electromagnetic fields, but also by torsion $\chi(t)$, created by spin of an electron. The last term in right hand side of equation (11) contain torsion χ, therefore *accelerated particle possessing a* *spin, radiates at the same time electromagnetic and electro-torsion fields (fields of Ricci torsion).* This theoretical conclusion is excellently confirmed by numerous experimental facts[9]. It is necessary to note that until now special experiments on research of structure of the reaction force of the radiation were not carried out. Only the surprising N.Tesla devices are known permitting to transmit electromagnetic energy by a way, not explained by conventional electrodynamics[10]. The system of the equations (8) (9) describes the motion of the origin of an orientable material point taking into account of its spin (torsion χ) and, certainly, generalizes the equations of Newton's mechanics.

III. INTERNAL SPACE OF THE ANHOLONOMIC ROTATIONAL COORDINATES AND ROTATIONAL RELATIVITY

During infinitesimal displacement of point M along the curve the triad of Frenet's vectors simultaneously change their orientation in space. For description of the change it is convenient to introduce anholonomic angular coordinates

$$\varphi = \angle(\mathbf{e}_1\ \mathbf{e}_\xi), \quad \psi = \angle(\mathbf{e}_\xi\ \mathbf{e}_{1'}), \quad \theta\angle(\mathbf{e}_3\ \mathbf{e}_{3'}),$$

$$(0 \le \varphi \le 2\pi, \quad 0 \le \psi \le 2\pi, \quad 0 \le \theta \le \pi,)$$

– Euler angles (see (fig.1a)). Let's assume, that with a point M of a curve the triad with motionless unit vectors $\mathbf{e}_1,\ \mathbf{e}_2,\ \mathbf{e}_3$ is connected. Let's designate components of motionless Frenet triad as

$$\mathbf{t} = \mathbf{e}_1^{'},\ \mathbf{n} = \mathbf{e}_2^{'},\ \mathbf{b} = \mathbf{e}_3^{'}.$$

At displacement of the origin O of Frenet triad along a curve from a point M in a point M', there is a rotation vectors of Frenet triad (fig. 2a). Projecting the axes of a mobile triad $\mathbf{t},\ \mathbf{n},\ \mathbf{b}$, located in a point M' on the motionless triad connected with a point M, we find

$$\mathbf{t} = \mathbf{e}_1^{'} = \mathbf{e}_1(\cos\varphi\cos\psi - \sin\varphi\sin\psi\cos\theta) + \mathbf{e}_2(\sin\varphi\cos\psi + \cos\varphi\sin\psi\cos\theta) + \mathbf{e}_3\sin\psi\sin\theta,$$

$$\mathbf{n} = \mathbf{e}_2^{'} = -\mathbf{e}_1(\cos\varphi\sin\psi + \sin\varphi\cos\psi\cos\theta) - \mathbf{e}_2(\sin\varphi\sin\psi - \cos\varphi\cos\psi\cos\theta) + \mathbf{e}_3\cos\psi\sin\theta,$$

$$\mathbf{b} = \mathbf{e}_3^{'} = \mathbf{e}_1\sin\varphi\sin\theta - \mathbf{e}_2\cos\varphi\sin\theta + \mathbf{e}_3\cos\theta.$$

Expressing the components of tangent vector $\mathbf{t} = \mathbf{e'}_1 = d\mathbf{x}/ds$ through angular variables, we have

$$\frac{dx}{ds} = \cos\varphi\ cos\psi - \sin\varphi\ \sin\psi\ \cos\theta, \tag{12}$$

$$\frac{dy}{ds} = \sin\varphi\ \cos\psi + \cos\varphi\ \sin\psi\ \cos\theta, \tag{13}$$

$$\frac{dz}{ds} = \sin\psi\ \sin\theta. \tag{14}$$

Differentiating the third components of vectors $\mathbf{t}$ and $\mathbf{n}$ and second component of the vector $\mathbf{b}$, we get "rotational equations of motion" as follows

$$\frac{d\varphi}{ds} = \chi\frac{\sin\psi}{\sin\theta}, \tag{15}$$

$$\frac{d\psi}{ds} = \kappa - \chi\ \sin\psi\ ctg\theta, \tag{16}$$

$$\frac{d\phi}{ds} = \chi\ \cos\psi. \tag{17}$$

According to the equations (15)-(17) curvature κ and torsion χ cause rotation of Frenet triad, therefore more correctly to name their first $\chi_1 = \kappa$ and second $\chi_2 = \chi$ torsion of a curve. The system of the equations (12)-(17) represents system Cauchy for six unknown functions $x,\ y,\ z,\ \varphi,\ \psi,\ \theta$ nd supposes one and only one solution in the form of regular functions

$$x = x(s),\ y = y(s),\ z = z(s),\ \varphi = \varphi(s)$$

$$,\ \psi = \psi(s),\ \theta = \theta(s)$$

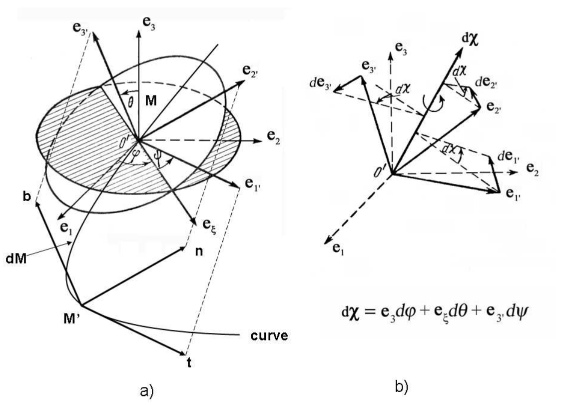

FIG. 2: Changing of the orientation of an oriented point at displacement of point M on dM; b) according to Euler's theorem an infinitesimal rotations around the three axes it is possible to replace by one rotation with the an infinitesimal angle.

satisfying to the equations (12)-(17) and entry conditions

$$x = x_0,\ y = y_0,\ z = z_0,\ \varphi = \varphi_0,\ \psi = \psi_0,\ \theta = \theta_0$$

for $s = s_0$. Entry conditions have simple geometrical sense. Initial coordinates $x = x_0,\ y = y_0,\ z = z_0$ define position of an original point M_0 a curve , and Euler's angles $\varphi = \varphi_0,\ \psi = \psi_0,\ \theta = \theta_0$ - initial orientation of the attached triad. Three Euler's angles form in each point M of a curve internal space anholonomic rotational coordinates, which, as it follows from the equations (12)-(17), define the physical dynamics of an orientable material point. Passing to the time parameter t in the equations (12)-(14), we will get

$$v_x(\varphi,\psi,\theta) = \frac{dx}{dt} = v(\cos\varphi\ cos\psi - \sin\varphi\ \sin\psi\ \cos\theta), \tag{18}$$

$$v_y(\varphi,\psi,\theta) = \frac{dy}{dt} = v(\sin\varphi\ \cos\psi + \cos\varphi\ \sin\psi\ \cos\theta), \tag{19}$$

$$v_z(\psi,\theta) = \frac{dz}{dt} = v(\sin\psi\ \sin\theta), \tag{20}$$

where$v = ds/dt$ – absolute velocity. Dependence of the components of linear velocity from Euler's angels in these equations allows us to approve, that the system of the equations (12)-(17) gives the analytical description of Descartes idea that any physical motion is rotation. Let's write down a Frenet triad as

$$e^{A}_{\ \alpha}\ ,$$

Where holonomic index α accepts values 1, 2, 3, and index A – local anholonomic index accepts values 1, 2, 3, designating numbers of a triad vectors. If on the holonomic coordinate index α the triad $e^{A}_{\ \alpha}$ has the tensor law of transformation in group of translations $T(3)$

$$e^{A}_{\ \alpha'} = \frac{\partial x_{\alpha'}}{\partial x_\alpha} e^{A}_{\ \alpha}, \quad \alpha = 1,2,3, \quad \|\frac{\partial x_{\alpha'}}{\partial x_\alpha}\| \in T(3), \tag{21}$$

that on the anholonomic local index A triad $e^{A}_{\ \alpha}$ it will be transformed in group of local three-dimensional rotations $O(3)$

$$e^{A'}_{\ \alpha} = \Lambda^{A'}_{\ A} e^{A}_{\ \alpha}, \quad A = 1,\ 2,\ 3, \quad \Lambda^{A'}_{\ A} \in O(3). \tag{22}$$

We see, that use of the rotational coordinates as elements of space of events generates a rotational relativity in nonrelativistic mechanics. This is one more fundamental distinction between Newton mechanics and the mechanics of an orientable point. At the description of motion of the Frenet triad the group $T(3)$ and $O(3)$T form Poincare's group with semidirect product of translations and rotations, as rotation of vectors of a triad causes translation of its origin – M points of M and vice versa. This fact substantially distinguishes mechanics of an orientable point from the Newtonian mechanics, as in the new mechanics translational and rotational momentums appear dependent from each other.

IV. CONNECTION OF $\chi_1(s)$ AND $\chi(s)_2$ WITH RICCI ROTATIONAL COEFFICIENTS AND ROTATIONAL METRIC

Statement 1. Torsion χ_1 and χ_2 are independent components of Ricci rotation coefficients. *Proof.* Let's consider six-dimensional manifold of coordinates $x_1, x_2, x_3, \varphi_1, \varphi_2, \varphi_3$. It is convenient to present it as a vector bundle with the base formed by translational coordinates x_1, x_2, x_3 (let it be Cartesian coordinates) and fibre, specified at each point x_α ($\alpha = 1,2,3$) by three orthonormalized Frenet's reference vectors

$$\mathbf{e}_A, \qquad A = 1,2,3, \tag{23}$$

where A means number of the reference vector. According to Euler's theorem, an infinitesimal rotations around the three axes of reference vector (23) is equivalent to one rotation with angle $\mathbf{d\chi}$ around a definite axis passing through the origin of the axis O (see (fig.2b) . It is possible to define the infinitesimal rotation as

$$\mathbf{d\chi} = d\chi \mathbf{e}_\chi,$$

where vector $\mathbf{e}_\chi$ is directed along instantaneous rotation axis of reference system. This direction is selected so that, if one looks from the end of the vector $\mathbf{e}_\chi$ e at a fixed point O, then the rotation is made counter-clockwise (right-hand reference system). Let's note, that the vector $\boldsymbol{\chi}$ does not exist, as turn on a finite angle is not commutative. Therefore for an infinitesimal rotation we have entered a designation $\mathbf{d}\boldsymbol{\chi}$ instead of $d\boldsymbol{\chi}$. Unlike a polar vector holonomic translational coordinates $d\mathbf{x}$, infinitesimal rotation

$$\mathbf{d}\boldsymbol{\chi} = \mathbf{e}_3 d\varphi + \mathbf{e}_\xi d\theta + \mathbf{e}_{3'} d\psi \tag{24}$$

is an axial vector. An infinitesimal rotation of Frenet's reference vectors $\mathbf{e}_\chi$ upon rotation $\mathbf{d}\boldsymbol{\chi}$ has the form

$$d\mathbf{e}_A = [\mathbf{d}\boldsymbol{\chi}\mathbf{e}_A]. \tag{25}$$

If we divide (25) by ds, then we shall get

$$\frac{d\mathbf{e}_A}{ds} = [\frac{\mathbf{d}\boldsymbol{\chi}}{ds}\mathbf{e}_A] = [\omega, \mathbf{e}_A], \tag{26}$$

where $\omega = \mathbf{d}\boldsymbol{\chi}/ds$ - three-dimensional angular velocity of Frenet's triad with respect to the instantaneous axis. Writing down the orthogonality conditions for Frenet's reference vectors in the form

$$a) \qquad e^A{}_\alpha e^\alpha_B = \delta^A_B = \begin{cases} 1 & A = B \\ 0 & A \neq B \end{cases}, \tag{27}$$

$$b) \qquad e^A{}_\alpha e^\beta{}_A = \delta_\alpha^{\ \beta} = \begin{cases} 1 & \alpha = \beta \\ 0 & \alpha \neq \beta \end{cases},$$

$$A, B... = 1, 2, 3, \qquad\qquad \alpha, \delta, \beta = 1, 2, 3,$$

where $\alpha, \delta, \beta...$ – holonomic coordinate indices, and $A, B...$ – anholonomic triad local indices; it is possible to write down relations (25) and (26) as follows

$$de^A{}_\alpha = d\chi^\beta{}_\alpha e^A{}_\beta, \tag{28}$$

$$\frac{de^A{}_\alpha}{ds} = \frac{d\chi^\beta{}_\alpha}{ds} e^A{}_\beta. \tag{29}$$

Multiplying (28) and (29) by $e^\beta{}_A$, we get

$$d\chi^\beta{}_\alpha = T^{\ \beta}_{\ \alpha\gamma} dx^\gamma, \tag{30}$$

$$\frac{de^A{}_\alpha}{ds} = T^\beta_{\alpha\gamma} \frac{dx^\gamma}{ds} e^A{}_\beta, \tag{31}$$

where we have defined the designation

$$T^\alpha_{\beta\gamma} = e^\alpha_A e^A{}_{\beta,\gamma} = -e^A_\beta e^\alpha{}_{A,\gamma}$$

$$, T_{\alpha\beta\gamma} = -T_{\beta\alpha\gamma}, \gamma = \frac{\partial}{\partial x^\gamma}. \tag{32}$$

The quantities (31)were first introduced by G.Ricci[4] and since then they have been called Ricci rotation coefficients. Using the orthogonality conditions (27) and the rule of transformation to local indices

$$T^A_{B\gamma} = e^A{}_\alpha T^\alpha_{\beta\gamma} e^\beta{}_B,$$

let's rewrite equations (31) in local indices

$$\frac{de^A{}_\alpha}{ds} = T^A{}_{B\gamma} \frac{dx^\gamma}{ds} e^B{}_\alpha. \tag{33}$$

Let's chose vectors $e^{(1)}{}_\alpha, e^{(2)}{}_\alpha$ and $e^{(3)}{}_\alpha$ so, that they coincide with Frenet's vectors, and thus the vector $e^{(1)}{}_\alpha = dx_\alpha/ds = t_\alpha$ satisfies the condition $t_\alpha t^\alpha = 1$. Then the equations (29) become the well-known Frenet's equations (3-5), in which

$$\kappa = \chi_1(s) = T^{(1)}{}_{(2)\gamma} \frac{dx^\gamma}{ds},$$

$$\chi = \chi_2(s) = T^{(2)}{}_{(3)\gamma} \frac{dx^\gamma}{ds}. \tag{34}$$

While deducing (3-5) from (29), we used the following relations

$$\frac{dx^\gamma}{ds} = e^\gamma{}_{(1)}, \quad e^\gamma_{(1)} e^{(1)}_\gamma = 1. \tag{35}$$

From the relations (31) it is clear, that in Frenet's equations curvature and torsion are expressed through components of Ricci rotation coefficients (28), that proves the Statement 1.

The Ricci rotation coefficients are the part of the connection of absolute parallelism geometry[5] and have an anti-symmetry on the two lower indices

$$T^\alpha{}_{[\beta\gamma]} = -\Omega^{..\alpha}_{\beta\gamma},$$

$$\Omega^{..\alpha}_{\beta\gamma} = -\frac{1}{2} e^\alpha_A (e^A{}_{\beta,\gamma} - e^A{}_{\gamma,\beta}), \tag{36}$$

which it is possible to call *Ricci torsion*. Let's note, ones more, that the curvature κ and torsion χ of Frenet's curve would be more correctly called the first and second torsion, as they are both expressed through components of Ricci torsion (36).

From (35) we can find $ds = e_\alpha{}^{(1)} dx^\alpha$ and

$$ds^2 = e_\alpha{}^{(1)} dx^\alpha e^\alpha{}_{(1)} dx_\alpha = dx^\alpha dx_\alpha = dx^2 + dy^2 + dz^2. \tag{37}$$

This translational metrics is set on group $T(3)$ of translational coordinates and defines geometry of the 3D euclidian space, in which the curve is embedded. Besides as follows from (24) and (33), on the group of rotational coordinates $O(3)$ the rotational metrics is define.

$$\boldsymbol{\chi}^2 = d\varphi^2 + d\psi^2 + d\theta^2 = d\chi^{\alpha}{}_{\beta} d\chi^{\beta}{}_{\alpha}$$

$$= T^{\alpha}_{\beta\gamma} T^{\beta}{}_{\alpha\delta} dx^{\gamma} dx^{\delta} = d\tau^2, \qquad (38)$$

This metrics addresses in zero if the first and second torsions (34) address to zero.

V. 4D ORIENTED POINT AND ABSOLUTE PARALLELISM GEOMETRY

A 3D orientable material point is mathematical representation of an arbitrary accelerated three-dimensional system of reference. Motion of such system of reference is described by six equations as it has six degrees of freedom. It would be possible to put and solve the problem on what geometry possess the space of events of an arbitrary accelerated 3D systems of reference. However, we consider as more important question — what geometry possess space of events of an arbitrary accelerated 4D systems of reference or, that is the same, what the space of events form the relative coordinates of the 4D orientable material points? It is in advance possible to tell, and it is obvious, that 4D an arbitrary accelerated system of reference has 10 degrees of freedom, therefore, for the description of its motion, it is necessary to use 10 coordinates. Leaning on experience, which we have received at the description of the dynamics of an arbitrary accelerated 3D system, we shall consider 10D coordinate space in which 4 translational coordinates $x_0 = ct,\ x_1 = x,\ x_2 = y, x_3 = z$ describe motion of the origin O 4D orientable point and 6 angular coordinates $\varphi_1 = \varphi,\ \varphi_2 = \psi,\ \varphi_3 = \theta,\ \varphi_4 = \vartheta_x,\ \varphi_5 = \vartheta_y,\ \varphi_6 = \vartheta_z$ describe change of its orientation.

Consider a four-dimensional differentiable manifold of 4D oriented points with translational coordinates x^i ($i = 0,1,2,3 \quad a = 0,1,2,3$). Whit each point of the manifold we connect four vectors $e^a{}_i$ ($i = 0,1,2,3$) and four covectors $e^j{}_b$ with the orthogonality conditions

$$e^a{}_i e^j{}_a = \delta^j_i, \quad e^a{}_i e^i{}_b = \delta^a_b. \qquad (39)$$

Anholonomic tetrad $e^a{}_i$ defines the metric tensor of the space

$$g_{ik} = \eta_{ab} e^a{}_i e^b_k, \eta_{ab} = \eta^{ab} = \mathrm{diag}(1\ -1\ -1\ -1) \quad (40)$$

and the translational Riemannian metric

$$ds^2 = \eta_{ab} e^a{}_i e^b_k dx^i dx^k = g_{ik} dx^i dx^k. \qquad (41)$$

Using the tensor (43), we can construct the Christoffel symbols

$$\Gamma^i_{jk} = \frac{1}{2} g^{im} (g_{jm,k} + g_{km,j} - g_{jk,m}). \qquad (42)$$

that transform following a nontensor law of transformation

$$\Gamma^{k'}_{j'i'} = \frac{\partial^2 x^k}{\partial x^{i'} \partial x^{j'}} \frac{\partial x^{k'}}{\partial x^k} + \frac{\partial x^i}{\partial x^{i'}} \frac{\partial x^j}{\partial x^{j'}} \frac{\partial x^{k'}}{\partial x^k} \Gamma^k_{ji} \qquad (43)$$

with respect to the coordinate transformations

$$dx^{i'} = \frac{\partial x^{i'}}{\partial x^k} dx^k, \quad \|\frac{\partial x^{i'}}{\partial x^k}\| \in T(4).$$

were $T(4)$ – group of 4D translations. Now the Ricci rotation coefficients (31) can be represented in the form

$$T^i_{jk} = e^i{}_a \nabla_k e^a{}_j, T^i_{jk} = -e^a{}_j \nabla_k e^i{}_a, T_{ijk} = -T_{jik}, \qquad (44)$$

where ∇_k is a covariant derivative with respect to the Christoffel Γ^i_{jk} symbols. The rotational metric in the new space can be written as

$$d\tau^2 = d\chi^a{}_b d\chi^b{}_a = T^a_{bn} T^b{}_{am} dx^k dx^m\ , \qquad (45)$$

$$i,\ j\ , k... = 0,1,2,3, \quad a,\ b,\ , c... = 0,1,2,3.$$

Let we have an arbitrary curve in four-dimensional Riemannian space with translational coordinates $x^i, (i = 0,1,2,3)$. Then the curve is defined by three scalar invariants $\chi_1,\ \chi_2\ \chi_3$, and in our case the four-dimensional Frenet's equations have

$$\frac{D e^{(0)}_k}{ds} = \chi_1 e^{(1)}_k, \qquad (46)$$

$$\frac{D e^{(1)}_k}{ds} = \chi_1 e^{(0)}_k + \chi_2 e^{(2)}_k, \qquad (47)$$

$$\frac{D e^{(2)}_k}{ds} = -\chi_2 e^{(1)}_k + \chi_3 e^{(3)}_k, \qquad (48)$$

$$\frac{D e^{(3)}_k}{ds} = -\chi_3 e^{(2)}_k. \qquad (49)$$

Here vectors $e^{(0)}_k, e^{(1)}_k, e^{(2)}_k$ and $e^{(3)}_k$ form a tetrad, and D is the absolute differential with respect to the four-dimensional Christoffel symbols (43).

Statement 2. Any curve of Riemannian space can be considered as the geodesics of space of absolute parallelism[5], with equations of the form

$$\frac{d^2 x^i}{ds^2} = -\Gamma^i{}_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} - T^i{}_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds}. \qquad (50)$$

Proof. Connection of absolute parallelism is defined as[3]

$$\Delta^{i}{}_{jk} = \Gamma^{i}{}_{jk} + T^{i}{}_{jk} = e^{i}_{a} e^{a}{}_{j,k} = -e^{a}{}_{j} e^{i}{}_{a,k}. \qquad (51)$$

These relations can be rewritten as follows

$$T^{i}{}_{jk} = e^{i}{}_{a} \nabla_{k} e^{a}{}_{j} = -e^{a}{}_{j} \nabla_{k} e^{i}{}_{a}, \qquad (52)$$

where ∇_k - covariant derivative with respect to Christoffel symbols. Multiplying equality (52) on $e^{a}{}_{i}$ ($e^{j}{}_{a}$) and using the orthogonality conditions (39) let's present (52) as follows

$$a)\ \nabla_{k} e^{a}{}_{j} = T^{a}_{bk} e^{b}{}_{j} \quad or \quad b)\ \nabla_{k} e^{i}{}_{a} = -T^{i}{}_{jk} e^{j}{}_{a}. \quad (53)$$

Multiplying (41a) and (41b) on dx^k/ds, we shall obtain

$$\frac{De^{a}{}_{j}}{ds} = T^{a}_{bk} e^{b}{}_{j} \frac{dx^{k}}{ds}. \qquad (54)$$

$$\frac{De^{i}{}_{a}}{ds} = -T^{i}{}_{jk} e^{j}{}_{a} \frac{dx^{k}}{ds}. \qquad (55)$$

Uncovering in equations (55) the absolute differential and supposing in them $e^{i}{}_{(0)} = dx^i/ds$, we shall obtain geodesics equations (50).

Changing in equations (54) indices on which there is a contraction, we find

$$\frac{De^{a}{}_{k}}{ds^2} = T^{a}_{bj} e^{b}{}_{k} \frac{dx^{j}}{ds}.$$

Choosing in these equations the Frenet's tetrad and writing down them component by component, we have

$$\frac{De^{(0)}_{\ \ k}}{ds^2} = T^{(0)}_{\ \ (1)j} e^{(1)}_{\ \ k} \frac{dx^{j}}{ds}, \qquad (56)$$

$$\frac{De^{(1)}_{\ \ k}}{ds^2} = T^{(1)}_{\ \ (0)j} e^{(0)}_{\ \ k} \frac{dx^{j}}{ds} + T^{(1)}_{\ \ (2)j} e^{(2)}_{\ \ k} \frac{dx^{j}}{ds}, \qquad (57)$$

$$\frac{De^{(2)}_{\ \ k}}{ds^2} = T^{(2)}_{\ \ (1)j} e^{(1)}_{\ \ k} \frac{dx^{j}}{ds} + T^{(2)}_{\ \ (3)j} e^{(3)}_{\ \ k} \frac{dx^{j}}{ds}, \qquad (58)$$

$$\frac{De^{(3)}_{\ \ k}}{ds^2} = T^{(3)}_{\ \ (2)j} e^{(2)}_{\ \ k} \frac{dx^{j}}{ds}. \qquad (59)$$

Comparing equations (65)-(68) with equations (77)-(80), we shall obtain

$$\chi_1 = T^{(0)}_{\ \ (1)j} \frac{dx^j}{ds}, \quad \chi_2 = T^{(1)}_{\ \ (2)j} \frac{dx^j}{ds}, \quad \chi_3 = T^{(2)}_{\ \ (3)j} \frac{dx^j}{ds}.$$

Since the quantities $T^{i}{}_{kj}$ are defined through Ricci torsion (see (52)), then, as it follows from relations obtained above, *is possible to geometrize any curves of Riemannian space, using Ricci torsion.*

The common symmetries os space of events of 4D oriented point are determined as:

a)by transformation of the four holonomic translation coordinates x_i , describing the motion of the origin of an arbitrary accelerated 4D frame

$$e^{a}{}_{i'} = \frac{\partial x_{i'}}{\partial x_i} e^{a}{}_{i}, \quad i = 0,1,2,3, \quad \|\frac{\partial x_{i'}}{\partial x_i}\| \in T(4), \ (60)$$

where $T(4)$ is a local group of 4D translations;

b) by transformation of the six anholonomic rotational coordinates $\boldsymbol{\chi}_{ab} = -\boldsymbol{\chi}_{ba}$, describing rotation of 4D oriented point (or an arbitrary accelerated 4D frame)

$$e^{a'}_{\ \ i} = \Lambda^{a'}_{\ \ a} e^{a}{}_{i}, \quad a = 0, 1,\ 2,\ 3, \quad \Lambda^{a'}_{\ \ a} \in O(1.3), \ (61)$$

where $O(1.3)$ is a local Lorenz group of 4D rotations. Term "local group" means, that the parameters of the group depends on the point of the curve.

The matrix $\Lambda^{a'}_{\ \ a}$ can be represented as

$$\Lambda^{a'}_{\ \ a} = R^{a'}_{\ \ b} L^{b}{}_{a}$$

where

$$R^{a'}_{\ \ b} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_{xx} & \cos\varphi_{xy} & \cos\varphi_{xz} \\ 0 & \cos\varphi_{yx} & \cos\varphi_{yy} & \cos\varphi_{yz} \\ 0 & \cos\varphi_{zx} & \cos\varphi_{zy} & \cos\varphi_{zz} \end{pmatrix}, \qquad (62)$$

is the matrix of the spatial rotations and

$$L^{b}{}_{a} = \begin{pmatrix} \gamma & -\beta_x\gamma & -\beta_y\gamma & -\beta_z\gamma \\ -\beta_x\gamma & 1+\frac{(\gamma-1)\beta_x^2}{\beta^2} & \frac{(\gamma-1)\beta_x\beta_y}{\beta^2} & \frac{(\gamma-1)\beta_x\beta_z}{\beta^2} \\ -\beta_y\gamma & \frac{(\gamma-1)\beta_x\beta_y}{\beta^2} & 1+\frac{(\gamma-1)\beta_y^2}{\beta^2} & \frac{(\gamma-1)\beta_y\beta_z}{\beta^2} \\ -\beta_z\gamma & \frac{(\gamma-1)\beta_x\beta_z}{\beta^2} & \frac{(\gamma-1)\beta_y\beta_z}{\beta^2} & 1+\frac{(\gamma-1)\beta_z^2}{\beta^2} \end{pmatrix}, \qquad (63)$$

– is the matrix, which describes rotation in space-time planes. Here

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2$$

– is relativistic factor, in which 3D velocity $v_\alpha = dx_\alpha/dt$ of 4D frame connects whit β_α and space-time angle ϑ_α as

$$\frac{v_\alpha}{c} = \beta_\alpha = th\vartheta_\alpha, \qquad (64)$$

were c - velocity of light.

VI. GENERALIZATION OF THE EINSTEIN'S MECHANICS

Einstein's General Relativity assumes the description of laws of physics in an arbitrary accelerated 4D frames. As we have shown, an arbitrary accelerated 4D frame has 10 digress of freedom an describes by 10 equations of motions: four equations of motions of the origin of 4D frame (50) and six rotational equations of motion (55). Einstein used only four equations

$$\frac{d^2x^i}{ds^2} = -\Gamma^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}. \qquad (65)$$

A. Generalization of the equations of motion of accelerated 4D frame

The equations of motion of the origin of 4D oriented point (or an arbitrary accelerated 4D frame)coincide with the equations of the geodesics of the space of absolute parallelism

$$\frac{d^2x^i}{ds^2} + \Gamma^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} + T^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0, \qquad (66)$$

which differ from the equations of motion in Einstein's theory of gravitation (65) by the additional term

$$T^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

The name of the quantities

$$T^i{}_{jk} = e^i{}_a \nabla_k e^a{}_j$$

–the Ricci rotation coefficients suggests that they describe rotation. It follows from our analysis, that the quantities $T^i{}_{jk}$ describe the change in the orientation of the tetrad vectors $e^a{}_j$ when the origin of tetrad shifts by an infinitesimal distance dx^i. Einstein interpreted symbols $\Gamma^i{}_{jk}$ in his equations (65) as intensity of a gravitational field. The object $\Gamma^i{}_{jk}$ get transformed relative to the transformations in $T(4)$ group as nontensor, whit respect formula (43). So, using normal coordinates, we can make $\Gamma^i{}_{jk}$ equal to zero. The Ricci rotational coefficients under transformation of translation coordinates in $T(4)$ transform as tensor

$$T^{k'}_{j'i'} = \frac{\partial x^i}{\partial x^{i'}}\frac{\partial x^j}{\partial x^{j'}}\frac{\partial x^{k'}}{\partial x^k}T^k_{ji} \qquad (67)$$

Writing down the equations (66) in normal coordinates, we have

$$\frac{d^2x^i}{ds^2} + T^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0, \qquad (68)$$

Using the Ricci rotation coefficients we can form the 4D angular velocity of the tetrad vector

$$\Omega^i{}_j = T^i{}_{jk}\frac{dx^k}{ds} \qquad (69)$$

with the symmetry properties

$$\Omega_{ij} = -\Omega_{ji}. \qquad (70)$$

Suppose now that the tetrad vectors coincide with the vectors of a 4D arbitrarily accelerated reference frame, then, by (69), the rotation of the reference frame is fully determined by the torsion field $T^i{}_{jk}$. Since the field $T^i{}_{jk}$ transforms following a tensor law relative to the coordinates transformations x_i, the rotation of reference frames relative to the coordinate transformations is absolute. The nontensor transformation law of $T^i{}_{jk}$ is valid for transformations in the angular coordinates φ_1, φ_2, φ_3, ϑ_1, ϑ_2, ϑ_3, therefore rotation is only relative for the group of rotations $O(1.3)$[5]. Let us now write the nonrelativistic equations of motion of a mass m under inertia forces alone, assuming that at a given moment of time it passes through the origin of an accelerated system

$$\frac{d}{dt}(m\mathbf{v}) = m(-\mathbf{W} + 2[\mathbf{v}\boldsymbol{\omega}]), \qquad (71)$$

where $-m\mathbf{W}$– force of inertia, arising at forward acceleration and $2m[\mathbf{v}\boldsymbol{\omega}]$ – Coriolis force of inertia.

These equations can be written in the form

$$\frac{d}{dt}(mv_\alpha) = m(-W_{\alpha o} + 2\omega_{\alpha\beta}\frac{dx^\beta}{dt}), \quad \alpha,\beta = 1,2,3, \qquad (72)$$

where $\mathbf{W} = (W_1, W_2, W_3) = (W_{10}, W_{20}, W_{30}), \boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$,

$$\omega_{\alpha\beta} = -\omega_{\beta\alpha} = -\begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}. \qquad (73)$$

On the other hand, equations (68), if we take into account (69), can be represented as

$$\frac{d^2x^i}{ds^2} + \Omega^i{}_j\frac{dx^j}{ds} = 0. \qquad (74)$$

Multiplying these equations by mass m, we will write the nonrelativistic three-dimensional part of these equations in the form

$$m\frac{du_\alpha}{ds_0} = -m\Omega_{\alpha 0}\frac{dx^0}{ds_0} - 2m\Omega_{\alpha\beta}\frac{dx^\beta}{ds_0}. \qquad (75)$$

Since in a nonrelativistic approximation $\mathrm{ds}_0 = cdt, u_\alpha = \frac{v_\alpha}{c} and \mathrm{dx}_0 = cdt$, then the equations (75) become

$$m\frac{dv_\alpha}{dt} = -mc^2\Omega_{\alpha 0} - 2mc^2\Omega_{\alpha\beta}\frac{1}{c}\frac{dx^\beta}{dt}. \tag{76}$$

Comparing (76) with (72) gives

$$\Omega_{10} = \frac{W_1}{c^2}, \quad \Omega_{20} = \frac{W_2}{c^2}, \Omega_{30} = \frac{W_3}{c^2}, \quad \Omega_{12} = -\frac{\omega_3}{c}, \Omega_{13} = \frac{\omega_2}{c}, \quad \Omega_{23} = -\frac{\omega_1}{c}.$$

Consequently, the matrix of the 4D angular velocity of rotation of an arbitrarily accelerated reference frame (matrix of the 4D "classical spin") has the form

$$\Omega_{ij} = \frac{1}{c^2}\begin{pmatrix} O & -W_1 & -W_2 & -W_2 \\ W_1 & 0 & -c\omega_3 & c\omega_2 \\ W_2 & c\omega_3 & 0 & -c\omega_1 \\ W_3 & -c\omega_2 & c\omega_1 & 0 \end{pmatrix} = \frac{1}{c}\begin{pmatrix} O & -\Theta_1 & -\Theta_2 & -\Theta_2 \\ \Theta_1 & 0 & -\omega_3 & \omega_2 \\ \Theta_2 & \omega_3 & 0 & -\omega_1 \\ \Theta_3 & -\omega_2 & \omega_1 & 0 \end{pmatrix}, \tag{77}$$

were $\omega_\alpha = d\varphi_\alpha/dt, \quad \alpha = 1,2,3$ – spatial angular velocity and $\Theta_\alpha = d\vartheta_\alpha/dt, \quad \alpha = 1,2,3$– angular velocity in the space-time planes. So, in nonrelativistic approximation 3D acceleration of 4D frame origin

$$W_\alpha = c\Theta_\alpha = c\frac{d\vartheta_\alpha}{dt}, \quad \alpha = 1,2,3$$

looks like rotation in the space-time planes. It is seen from the matrix that the 4D rotation of a frame caused by the inertial fields $T^i{}_{jk}$ is associated with the torsion

$$\Delta^i{}_{[jk]} = T^i{}_{[jk]} = -\Omega_{jk}^{\cdot\cdot i} = -e^i{}_a e^a{}_{[k,j]} = -\frac{1}{2}e^i{}_a(e^a{}_{k,j} - e^a{}_{j,k}) \tag{78}$$

of a space of absolute parallelism, since

$$T^i{}_{jk} = -\Omega_{jk}^{\cdot\cdot i} + g^{im}(g_{js}\Omega_{mk}^{\cdot\cdot s} + g_{ks}\Omega_{mj}^{\cdot\cdot s}). \tag{79}$$

Fields determined by the rotation of space came to be known as torsion fields. Accordingly, the torsion field $T^i{}_{jk}$ represents the inertial field engendered by the torsion of a space of absolute parallelism[11].

B. Generalization of the Einstein' vacuum equations

An empty, but curved space in Einstein's theory obeys the equations

$$R_{ik} = 0, \tag{80}$$

whose Schwarzschild's solution is supported by experiment (the shift of Mercury's perihelion, the deviation of a light ray in the Solar gravitational field, the delay of radiosignals in a gravitational field, etc.).

Note that Einstein's vacuum equations *do not contain any physical constants*. They are purely field nonlinear equations, and Einstein held that a correct generalization of these equations would lead us to equations of the unified field theory. He wrote[6]: "I believe, further, that the equations of gravitation for empty space are the only rationally justified case of field theory that can claim to be rigorous (considering nonlinear terms as well). This all leads to an attempt to generalize the gravitation theory for empty space."

Einstein believed that one of the main problems in unified field theory is the one of the geometrization of the energy-momentum tensor of matter on the right-hand side of his equations (1). This problem can be solved using the concept of 4D oriented point and the space of events with the geometry of absolute parallelism and Cartan's structural equations in this geometry[5]:

Displacement of the origin and changing of the orientation of 4D oriented point can be presented by differentials

$$dx^i = e^a e^i{}_a, \tag{81}$$

$$de^i{}_b = \Delta^a{}_b e^i{}_a, \tag{82}$$

where

$$e^a = e^a_i dx^i, \tag{83}$$

$$\Delta^a{}_b = e^a{}_i de^i{}_b = \Delta^a{}_{bk} dx^k \tag{84}$$

are differential 1-forms of tetrad $e^a{}_i$ and connection of absolute parallelism $\Delta^a{}_{bk}$. Differentiating the relationships (81), (82) externally, we have, respectively,

$$d(dx^i) = (de^a - e^c \wedge \Delta^a{}_c)e^i{}_a = -S^a e^i{}_a, \tag{85}$$

$$d(de^i{}_a) = (d\Delta^b{}_a - \Delta^c{}_a \wedge \Delta^b{}_c)e^i{}_b = -S^b{}_a e^i{}_b. \tag{86}$$

Here S^a denotes the 2-form of Cartanian torsion, and $S^b{}_a$ – the 2-form of the curvature tensor. The sign $\wedge$ signifies external product, e.g,

$$e^a \wedge e^b = e^a e^b - e^b e^a. \tag{87}$$

By definition, a space has a geometry of absolute parallelism, if the 2-form of Cartanian torsion S^a and the 2-form of the Riemann-Christoffel curvature $S^b{}_a$ of this space vanish

$$S^a = 0, \tag{88}$$

$$S^b{}_a = 0. \tag{89}$$

At the same time, these equalities are the integration conditions for the differentials (88) and (89).

Equations

$$de^a - e^c \wedge \Delta^a{}_c = -S^a, \tag{90}$$

$$d\Delta^b{}_a - \Delta^c{}_a \wedge \Delta^b{}_c = -S^b{}_a, \tag{91}$$

which follow from (85) and (86), are Cartan's structural equations for an appropriate geometry. For the geometry of absolute parallelism hold the conditions (88) and (89), therefore Cartan's structural equations for A_4 geometry have the form

$$de^a - e^c \wedge \Delta^a{}_c = 0, \tag{92}$$

$$d\Delta^b{}_a - \Delta^c{}_a \wedge \Delta^b{}_c = 0. \tag{93}$$

Considering (51), we will represent 1-form $\Delta^a{}_b$ as the sum

$$\Delta^a{}_b = \Gamma^a{}_b + T^a_{\ b}. \tag{94}$$

Substituting this relationship into (92) and noting that

$$e^c \wedge \Delta^a{}_c = e^c \wedge T^a_{\ c},$$

we get the first of Cartan's structural equations for space of events of the 4D oriented points

$$de^a - e^c \wedge T^a_{\ c} = 0.$$

In matrix form these equations will look like

$$\nabla_{[k} e^a{}_{m]} - e^b{}_{[k} T^a_{\ |b|m]} = 0. \tag{A}$$

Substituting (94) into (93) gives the second of Cartan's equations for the space.

$$R^a_b + dT^a_{\ b} - T^c_{\ b} \wedge T^a_{\ c} = 0,$$

or, in matrix form

$$R^a{}_{bkm} + 2\nabla_{[k} T^a_{\ |b|m]} + 2T^a_{\ c[k} T^c_{\ |b|m]} = 0. \tag{B}$$

In the coordinate indexes the equations (B), written as

$$R^i{}_{jkm} + 2\nabla_{[k} T^i_{\ |j|m]} + 2T^i_{\ s[k} T^s_{\ |j|m]} = 0. \tag{95}$$

Forming, using (95), the Einstein tensor

$$G_{jm} = R_{jm} - \frac{1}{2} g_{jm} R,$$

we obtain the 10 equations

$$R_{jm} - \frac{1}{2} g_{jm} R = \nu T_{jm}, \tag{96}$$

which are similar to Einstein's equations, but with the geometrized right-hand side defined as

$$T_{jm} = -\frac{2}{\nu}\{(\nabla_{[i} T^i{}_{|j|m]} + T^i{}_{s[i} T^s{}_{|j|m]}) - \frac{1}{2} g_{jm} g^{pn} (\nabla_{[i} T^i{}_{|p|n]} + T^i{}_{s[i} T^s{}_{|p|n]}) \tag{97}$$

Let us now decompose the Riemann tensor R_{ijkm} into irreducible parts

$$R_{ijkm} = C_{ijkm} + g_{i[k} R_{m]j} + g_{j[k} R_{m]i} + \frac{1}{3} R g_{i[m} g_{k]j}, \tag{98}$$

where C_{ijkm} is the Weyl tensor; the second and third terms are the traceless part of the Ricci tensor R_{jm} and R is its trace.

Using the equations (96), written as

$$R_{jm} = \nu \left(T_{jm} - \frac{1}{2} g_{jm} T \right), \tag{99}$$

we will rewrite the relationship (98) as

$$R_{ijkm} = C_{ijkm} + 2\nu g_{[k(i} T_{j)m]} - \frac{1}{3} \nu T g_{i[m} g_{k]j}, \tag{100}$$

where T is the tensor trace (97).

Now we introduce the tensor current

$$J_{ijkm} = 2 g_{[k(i} T_{j)m]} - \frac{1}{3} T g_{i[m} g_{k]j} \tag{101}$$

and represent the tensor (100) as the sum

$$R_{ijkm} = C_{ijkm} + \nu J_{ijkm}. \tag{102}$$

Substituting this relationship into the equations (95), we will arrive at

$$C_{ijkm} + 2\nabla_{[k} T_{|ij|m]} + 2 T_{is[k} T^{s}_{|j|m]} = -\nu J_{ijkm}. \tag{103}$$

Equations (103) are the Yang-Mills equations with a geometrized source, which is defined by the relationship (101). In equations (103) for the Yang-Mills field we have the Weyl tensor C_{ijkm}, and the potentials of the Yang-Mills field are the Ricci rotation coefficients T^{i}_{jk}. Summarizing the geometrized Einstein equations (96) and the Yang-Mills equations (103), we can represent the structural Cartan equations (A) and (B) as an extended set of Einstein-Yang-Mills equations

$$\nabla_{[k} e^{a}_{j]} + T^{i}_{[kj]} e^{a}{}_{i} = 0, \tag{A}$$

$$R_{jm} - \frac{1}{2} g_{jm} R = \nu T_{jm}, \tag{$B.1$}$$

$$C^{i}_{jkm} + 2\nabla_{[k} T^{i}_{|j|m]} + 2 T^{i}_{s[k} T^{s}_{|j|m]} = -\nu J^{i}{}_{jkm}, \tag{$B.2$}$$

in which the geometrized sources T_{jm} and J_{ijkm} are given by (97) and (101).

For the case of Einstein's vacuum the equations are much simpler

$$\nabla_{[k} e^{a}_{j]} + T^{i}_{[kj]} e^{a}{}_{i} = 0, \tag{i}$$

$$R_{jm} = 0, \tag{ii}$$

$$C^{i}{}_{jkm} + 2\nabla_{[k} T^{i}_{|j|m]} + 2 T^{i}_{s[k} T^{s}_{|j|m]} = 0. \tag{iii}$$

Thus, the structural equations of absolute parallelism geometry, represent an extended set of Einstein-Yang-Mills equations with the gauge translations group T_4 defined on the base x^i with the structural equations (A), and with the gauge rotational group $O(1.3)$, defined in the fibre $e^{i}{}_{a}$ with the structural equations in the form of the geometrized Einstein-Yang-Mills equations $(B.1)$ and $(B.2)$.

It is easy to see, that when torsion $\Omega_{jk}{}^{i}$ (and , hence, torsion field $T^{i}{}_{jk}$) in the (A) and (B) equations is equal to zero the space of events becomes Minkovski space. The converse proposition, generally, is incorrect. If to put in the equations (A) and (B) Riemannian curvature equal to zero, we shall receive the equations

$$\nabla_{[k} e^{a}_{j]} + T^{i}_{[kj]} e^{a}{}_{i} = 0, \tag{104}$$

$$\nabla_{[k} T^{i}_{|j|m]} + T^{i}_{s[k} T^{s}_{|j|m]} = 0, \tag{105}$$

which describe so-called primary torsion fields[5].

VII. CORRESPONDENCE WITH THE EQUATIONS OF EINSTEIN'S THEORY

The equations (66) will be transformed to the equations of motions of Einstein's theory of gravitation when the inertia force in (66) becomes zero

$$F^{i}_{I} = m T^{i}{}_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0, \tag{106}$$

or, using (79) (for $m \neq 0$)

$$-\Omega_{jk}{}^{i} \frac{dx^j}{ds} \frac{dx^k}{ds} + g^{im} (g_{gs} \Omega_{mk}{}^{s} + g_{ks} \Omega_{mj}{}^{s}) \frac{dx^j}{ds} \frac{dx^k}{ds} = 0. \tag{107}$$

Since Ω_{mkj} is skew-symmetric in indices m and k, then it follows from (107) that in inertial reference frames the torsion Ω_{mkj} of the space is skew-symmetrical in all the three indices

$$T_{ijk} = -T_{jik} = -T_{ikg} = -\Omega_{ijk}, \tag{108}$$

but not equal to zero and coincides with torsion field $T^{i}{}_{jk}$. The energy-momentum tensor (97) in these case is symmetrical in the indices j and m to yield

$$T_{jm} = \frac{1}{\nu} (\Omega_{sm}{}^{i} \Omega_{ji}{}^{s} - \frac{1}{2} g_{jm} \Omega_{s}{}^{ji} \Omega_{ji}{}^{s}). \tag{109}$$

In general case torsion $\Omega_{jk}{}^{i}$ has 24 independent components and it can be represented as the sum of three irreducible parts as follows

$$\Omega^{i}_{\cdot jk} = \frac{2}{3} \delta^{i}{}_{[k} \Omega_{j]} + \frac{1}{3} \varepsilon^{n}{}_{jks} \hat{\Omega}^{\hat{s}} + \bar{\Omega}^{i}_{\cdot jk}, \tag{110}$$

where

$$\Omega^{i}_{\cdot jk} = g^{im} g_{ks} \Omega_{mj}^{\cdot\cdot s}, \quad (111)$$

and Ω_j – the vector, $\hat{\Omega}_j$ – the pseudovector and $\bar{\Omega}^{\ \ i}_{\cdot jk}$ –the traceless part of torsion are given by

$$\Omega_j = \Omega^{i}_{\cdot ji}, \quad (112)$$

$$\hat{\Omega}_j = \frac{1}{2}\varepsilon_{jins}\Omega^{ins}, \quad (113)$$

$$\bar{\Omega}^{s}_{\cdot js} = 0, \quad \bar{\Omega}_{ijs} + \bar{\Omega}_{jsi} + \bar{\Omega}_{sij} = 0, \quad (114)$$

where ε_{ijkm} is a fully skew-symmetrical Levi-Civita symbol.

Since in inertial reference frames the torsion Ω_{ijs} is skew-symmetrical in all the three indices, among the irreducible parts of torsion in inertial frames only the pseudovector (113) is nonzero.

We can define the auxiliary pseudovector h_m through the field (113) as follows

$$\Omega^{ijk} = \varepsilon^{ijkm} h_m, \quad \Omega_{ijk} = \varepsilon_{ijkm} h^m \quad (115)$$

and write the tensor (109) as

$$T_{jm} = \frac{1}{2\nu}(h_j h_m - \frac{1}{2} g_{jm} h^i h_i). \quad (116)$$

If the pseudovector h_m is light-like, it can be represented as

$$h_m = \Phi l_m, \quad l_m l^m = 0, \quad \Phi = \Phi(x^i). \quad (117)$$

In this case the matter tensor (116) becomes

$$T_{jm} = \frac{1}{\nu}\Phi^2(x^i) l_j l_m, \quad (118)$$

and the density of matter is given by

$$\rho = \frac{1}{\nu c^2}\Phi^2(x^i). \quad (119)$$

If the pseudovector h_m is time-like, it can conveniently be represented as

$$h_m = \varphi(x^i) u_m, \quad (120)$$

where

$$u_m u^m = 1 \quad (121)$$

and $\varphi(x^i)$ is a scalar quantity.

Substitution of (120) into the tensor (116) yields the energy-momentum tensor of the form

$$T_{jm} = \frac{1}{\nu}\varphi^2(u_j u_m - \frac{1}{2} g_{jm}) = -\rho c^2 (u_j u_m - \frac{1}{2} g_{jm}), \quad (122)$$

were

$$\rho = -\frac{1}{\nu c^2}\varphi^2(x^i) \quad (123)$$

density of the matter. Tensor (118) looks like an energy-momentum tensor *of isotropic radiation*, and the tensor (122) in its structure looks rather like the energy-momentum tensor *of an ideal liquid.* Thus, in a post-Einstein's approximation the matter density is defined through squares of torsion fields Φ and φ according to (119) and (123).

A. Motion of the torsion matter

The left-hand side of Einstein like equations (B.1) is always symmetrical in indices j and m, therefore these equations can be written as

$$R_{jm} - \frac{1}{2} g_{jm} R = \nu T_{(jm)}, \quad (124)$$

$$T_{[jm]} = \frac{1}{\nu}(-\nabla_i \Omega_{jm}^{\cdot\cdot i} - \nabla_m A_j - A_s \Omega_{jm}^{\cdot\cdot s}) = 0, \quad (125)$$

where

$$A_j = T^{i}{}_{ji}, \quad (126)$$

The equality (**??**) should be considered as a constraint which apply on the torsion $\Omega_{jk}^{\cdot\cdot i}$. At transition to the Einstein theory the condition(108) is satisfied, thus (**??**) vanish and from (**??**) we find

$$\nabla_i \Omega_{jm}^{\cdot\cdot i} = 0. \quad (127)$$

Substituting (115) into (127) gives

$$h_{m,j} - h_{j,m} = 0.$$

These equations have two solutions: $h_m = 0$ (a trivial one), and

$$h_m = \Psi_{,m},$$

where Ψ is pseudoscalar. Writing the energy-momentum tensor (127) through this pseudoscalar, we have

$$T_{jm} = \frac{1}{2\nu}(\Psi_{,j}\Psi_{,m} - \frac{1}{2} g_{jm}\Psi^{,i}\Psi_{,i}). \quad (128)$$

In quantum field theory the tensor (131) is the energy-momentum tensor of a massless pseudoscalar field, where the pseudoscalar Ψ plays the role of the wave function in quantum equations of motion.

According to the field equations (124), torsion $\Omega_{jm}^{\cdot\cdot i}$ "tells" geometry how to "curve"; furthermore, from the field equations (124) itself, geometry "tells" matter how to move. Using second Bianchi's identities for Riemann tensor, we can fined

$$\nabla_j(R^{jm} - \frac{1}{2}Rg^{jm}) = 0.$$

Applying this equality to the equations (124), we shall receive conservation law of the energy-momentum tensor from which the equations of motion follows

$$\nabla^j(R_{jm} - \frac{1}{2}g_{jm}R) = \nu\nabla^j T_{jm} = 0, \qquad (129)$$

from which the equations of motion follows.

The right-hand side of (129) yields for tensor (122) the equations of motion of the matter in the form

$$0 = \nabla^j T_{jm} = -\nabla_j(\rho c^2 u^j u^m) + \frac{1}{2}\nabla_j(\rho c^2 g^{jm}) = -\rho c^2 u^j \nabla_j u^m - c^2 u^m \nabla_j \rho u^j + \frac{1}{2}\rho c^2 \nabla_j g^{jm} + \frac{1}{2}g^{jm} c2\nabla_j \rho. \qquad (130)$$

The first term in the right part of this equality is equal to zero

$$-\rho c^2 u^j \nabla_j u^m = -\rho c^2(\frac{du^m}{ds} + \Gamma^m_{kn} u^k u^n) = 0, \qquad (131)$$

as this expression describes geodesic motion in the Einstein's theory The third term also equal to zero, because

$$\nabla_j g^{jm} = 0.$$

For an incompressible fluid we have

$$\nabla_j \rho = 0,$$

therefore from (122) we shall receive the equation of continuity

$$\nabla_j(\rho u^j) = \partial_j(\rho u^j) + \rho u^k \Gamma^j{}_{kj} = 0. \qquad (132)$$

In normal (local) coordinates $\Gamma^j{}_{kj} = 0$ and the equation (132) becomes

$$\frac{\partial \rho}{\partial t} + div(\rho \mathbf{v}) = 0. \qquad (133)$$

If to substitute here, for example, the density (123) we shall receive for function φ the nonlinear equation

$$\frac{\partial \varphi^2}{\partial t} + div(\varphi^2 \mathbf{v}) = 0.$$

B. Definition of ν factor in the field equations

We will consider the spherically symmetrical solution of the vacuum equations (A) and (B), which describe the vacuum excitation with a variable Newton potential and for which the Energy-momentum tensor (97) is different from zero. This solution has the following characteristics[5]:

1. Coordinates $x^0 = u, x^1 = r, x^2 = \theta, x^3 = \varphi$.
2. Components of the Newman-Penrose symbols

$$\sigma^i_{0\dot{0}} = (0,1,0,0), \quad \sigma^i_{1\dot{1}} = (1,U,0,0), \quad \sigma^i_{0\dot{1}} = \rho(0,0,P,iP),$$

$$\sigma_i^{0\dot{0}} = (1,0,0,0), \quad \sigma_i^{1\dot{1}} = (-U,1,0,0), \quad \sigma_i^{0\dot{1}} = -\frac{1}{2\rho P}(0,0,1,i)$$

$$U(u) = -1/2 + \Psi^0(u)/r, \quad P = (2)^{-1/2}(1+\zeta\bar{\zeta}/4), \quad \zeta = x^2 + ix^3,$$

$$\Psi^0 = \Psi^0(u).$$

3. Spinor components of the torsion field

$$\rho = -1/r, \quad \alpha = -\overline{\beta} = -\alpha^0/r, \quad \gamma = \Psi^0(u)/2r^2,$$

$$\mu = -1/2r + \Psi^0(u)/r^2, \quad \alpha^0 = \zeta/4.$$

4. Spinor components of the Riemann tensor

$$\Psi_2 = \Psi = -\Psi^0(u)/r^3, \quad \Phi_{22} = \Phi = -\dot{\Psi}^0(u)/r^2 = -\frac{\partial \Psi^0}{\partial u}\frac{1}{r^2}.$$

The Riemann metric of the solution (**??**) in (quasi) spherical coordinates has the form

$$ds^2 = \left(1 - \frac{2\Psi^0(t)}{r}\right) c^2 dt^2 - \left(1 - \frac{2\Psi^0(t)}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{134}$$

Using the solution (**??**), we can determine the explicit form of the energy-momentum tensor (97). Calculations will show that the tensor is

$$T_{jm} = \rho c^2 l_j l_m, \tag{135}$$

where ρ is the matter density of a vacuum excitation given by

$$\rho = -\frac{2\dot{\Psi}^o(u)}{\nu c^2 r^2}, \quad \dot{\Psi}^o(u) < 0 \tag{136}$$

and l_m is the light like vector $l_m l^m = 0$.

We now consider the limiting process $\Psi^o(u) \to \Psi^o = const$ of the matter density in the solution (**??**). We introduce the auxiliary parameter ξ with the dimensionality of length

$$\xi = \frac{\pi|\dot{\Psi}^o|r^2}{2\Psi^o}. \tag{137}$$

Through the parameter ξ the density module (136) can be represented as $\rho = \rho^+$

$$\rho = \frac{8\pi\Psi^o}{\nu c^2}\frac{1}{2\pi r^2}\frac{\xi}{r^2} = \frac{8\pi\Psi^o}{\nu c^2}\frac{1}{2\pi r^2}\frac{\xi}{(r^2+\xi^2)}\left(1 + \frac{\xi^2}{r^2}\right), \tag{138}$$

where the + sign implies that the density ρ^+ defines right-hand matter with a positive density and positive mass. Taking the limit in (138) for $\xi \to 0$, i.e., for $\Psi^o(u) \to \Psi^o = const$, and using the well-known formula

$$\frac{1}{2\pi r^2}\frac{1}{\pi}\lim_{x\to 0}\left(\frac{x}{x^2+r^2}\right) = \frac{1}{2\pi r^2}\delta(r) = \delta(\mathbf{r}),$$

where $\delta(\mathbf{r})$ is the three-dimensional Dirac function, we will get

$$\rho^+ = \frac{8\pi\Psi^o}{\nu c^2}\frac{1}{2\pi r^2}\delta(r) = \frac{8\pi\Psi^o}{\nu c^2}\delta(\mathbf{r}) = M\delta(\mathbf{r}), \tag{139}$$

where $M = const$ - mass of a point source.

On the other hand, as the source goes stationary, the metric (134) becomes the Schwarzshield metric

$$ds^2 = (1 - \frac{2MG}{rc^2})c^2dt^2 - (1 - \frac{2MG}{rc^2})^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{140}$$

(i.e., the solution of Einstein's equations for a point source) provided that

$$\Psi^0 = \frac{MG}{c^2} = const. \tag{141}$$

Substituting (141) into the equality (139), we will obtain the value of initially arbitrary factor ν in the vacuum equations (B.1)

$$\nu = \frac{8\pi G}{c^4}. \tag{142}$$

In that case the equations (B.1) coincide with Einstein's equations that describe the gravitational field of a point source with constant mass M. It is seen from this relationship that when a vacuum excitation becomes stationary the matter density distributed over space coincides with the matter density for a point particle (Dirac's δ-function describes the distribution of a point source). The fact that a material point appears in a purely field theory as a limiting stationary case is one of the most important results of the new theory.

VIII. CONCLUSION

More than 300 years we have been applying Newton's mechanics to explain non-relativistic mechanical experiments. Although Newton's mechanics has been generalized three times: by the special relativity theory, general relativity theory, and quantum mechanics, there remains a possibility for its further generalization. The fourth generalization of Newtonian mechanics has become possible with regards that new mechanics has been based upon the following: 1) Clifford-Einstein program for geometrization of all physics equations, including classical mechanics, (Unified Field Theory[2]); 2) Cartan's idea about the connection of the torsion of space with physical rotation[7].

Einstein assumed the solution of these problems in the geometrization of the right hand of its equations. Generalizing Einstein's vacuum equations, we have introduce structural Cartan equations geometry of absolute parallelism as the new vacuum equations. It has allowed us not only to find a general view of geometrized energy-momentum tensor, but also to specify connection torsion of the space of absolute parallelism with a field of inertia. The mass of any object in the generalized theory has purely field nature and is defined as a measure of field of inertia. The rest mass of such object can be operated, using rotation of masses of which the object consists. The first experimental acknowledgement of these theoretical conclusions are already received by us at research of the dynamics so called 4D gyroscope[8].

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11. In the article " Riemann-Geometrie mit Aufrechterhaltung des Bergiffes des Fernaparralelismus" Sitzungsber. preuss. Akad.Wiss., phys.-math. Kl., 1928, 217-221 A.Einstein used the torsion of absolute parallelism determining torsion as

$$\Lambda^{i}{}_{jk} = \frac{1}{2}(\Delta^{i}{}_{jk} - \Delta^{i}{}_{kj}),$$

where

$$\Delta^{i}{}_{jk} = e^i_a e^a_{j,k}, \quad ,k = \frac{\partial}{\partial x_k}, \quad i, j, k... = 0, 1, 2, 3, \quad a, b... = 0, 1, 2, 3$$

–connection of absolute parallelism,

$$\Lambda^{i}{}_{jk} = -\Omega_{jk}^{\cdot\cdot\, i}$$

– anholonomity object in J. Schouten definition. In the same article A. Einstein has specified, that when torsion $\Lambda^{i}{}_{jk}$ (anholonomity object) is equal to zero the space becomes Minkovski space.